CONTEMPORARY MATHEMATICS

730

Model Theory of Modules, Algebras and Categories

International Conference
Model Theory of Modules, Algebras and Categories
July 28–August 2, 2017
Ettore Majorana Foundation and Centre for Scientific Culture, Erice, Sicily, Italy

Alberto Facchini
Lorna Gregory
Sonia L'Innocente
Marcus Tressl
Editors

AMS American Mathematical Society

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Contents

Preface v

List of Participants vii

On isonoetherian and isoartinian modules
Alberto Facchini and Zahra Nazemian 1

Derived categories for Grothendieck categories of enriched functors
Grigory Garkusha and Darren Jones 23

Left determined morphisms and free realisations
Lorna Gregory 47

The universal abelian regular ring
Ivo Herzog and Sonia L'Innocente 57

A characterisation of τ-tilting finite algebras
Lidia Angeleri-Hügel, Frederik Marks, and Jorge Vitória 75

Describing models of Th($\mathbb{Z}$) in adelic terms
Angus Macintyre 91

Valued modules on skew polynomial rings and Bézout domains
Françoise Point 97

Multisorted modules and their model theory
Mike Prest 115

Pure projective modules over non-singular serial rings
Pavel Příhoda 153

Mittag-Leffler modules and definable subcategories
Philipp Rothmaler 171

Intrinsic valuation entropy
Luigi Salce and Simone Virili 197

Decidability and modules over Bézout domains
Carlo Toffalori 223

Preface

This volume consist of 12 papers by invited speakers of the International Conference "Model Theory of Modules, Algebras and Categories", held at the *Ettore Majorana Foundation and Centre for Scientific Culture* in Erice, Italy, July 28–August 2, 2017. The meeting had 38 participants and was dedicated to Mike Prest of The University of Manchester on the occasion of his 65th birthday. The focus of the articles is on recent developments in the model theory of modules, as well as the broad influence, sometimes explicit, sometimes implicit, that the model theory of modules has had on the representation theory of algebras and certain regions of category theory. For over three decades now, the model theory of modules has developed under the auspices of Mike Prest and his school. The interdisciplinary nature of his work is reflected in this volume, with articles ranging from the following broad topics:

- Logic, notably model theory of modules and groups.
- Representation theory (pure-injective and pure-projective modules and their generalizations, silting theory).
- Category theory and homological algebra (derived categories, localizations, recollments).

The meeting was marked by the unexpected death of Gena Puninski in spring 2017. Gena was a leading figure in the model theory of modules and a collaborator and friend of many of the participants. On the second day of the conference a memorial session was held in honour of Gena.

We thank the speakers and participants who made the conference a success, all authors for their contributions and the referees for their time, careful reading and suggestions for improvement.

We gratefully acknowledge the material support for the conference from the following institutions: The School of Statistical Physics at the Ettore Majorana Foundation and Centre for Scientific Culture (in particular its director Professor Fabio Marchesoni) for their hospitality; The Tullio Levi-Civita Mathematics Department (University of Padua); The School of Science and Technology (University of Camerino); GNSAGA-INDAM, National Group for Algebraic and Geometric Structures and their Applications; Italian FIRB 2010 "New advances in Model Theory of exponentiation"; Italian PRIN2012 "Models and Sets"; The School of Mathematics (The University of Manchester).

Special thanks go to Christine Thivierge, of the AMS Contemporary Mathematics series, for her support in the preparation of this volume.

Alberto Facchini
Lorna Gregory
Sonia L'Innocente
Marcus Tressl
September 2018

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Model Theory of Modules, Algebras and Categories • Facchini et al., Editors

This volume contains the proceedings of the international conference Model Theory of Modules, Algebras and Categories, held from July 28–August 2, 2017, at the Ettore Majorana Foundation and Centre for Scientific Culture in Erice, Italy.

Papers contained in this volume cover recent developments in model theory, module theory and category theory, and their intersection.

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