SUPPLEMENTARY INFORMATION
Multicomponent electron-hole superfluidity and the BCS-BEC crossover in double bilayer graphene

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We use a standard multiband Hamiltonian,

$$H = \sum_{k\gamma} \left\{ \xi_k^{(e)\gamma}\, c_k^{\gamma\dagger}\, c_k^{\gamma} + \xi_k^{(h)\gamma}\, d_k^{\gamma\dagger}\, d_k^{\gamma} \right\} + \sum_{k,k',q,\gamma,\gamma'} V_{k\,k'}\, c_{k+q/2}^{\gamma\dagger}\, d_{-k+q/2}^{\gamma\dagger}\, d_{-k'+q/2}^{\gamma'}\, c_{k'+q/2}^{\gamma'} \,. \tag{S1}$$

The index $\gamma = \pm$ labels the conduction and valence bands of the electron and hole bilayer sheets. We make the standard transformation so the bands of the p-doped bilayer are filled with positively charged holes up to the Fermi level located in the conduction band. $c_k^{\gamma\dagger}$, $d_k^{\gamma\dagger}$ and c_k^{γ}, d_k^{γ} are the creation and destruction operators for the electrons, holes in their respective bilayer sheets (see Fig. S1). Spin indices are left implicit. $V_{k,k'}$ is the electron-hole interaction. The energy $\xi_k^{(e,h)\gamma} = \varepsilon_k^{\gamma} - \mu$, with the parabolic single-particle energy dispersion of the conduction and valence bands for the two bilayer graphene sheets: $\varepsilon_k^{+} = \hbar^2 k^2/2m^*$ and $\varepsilon_k^{-} = -\hbar^2 k^2/2m^* - E_g$, with equal effective masses for the electrons and holes, $m^* = 0.04 m_e$.[1] E_g is the bandgap between the conduction and valence bands which we take equal in the two sheets. We consider only equal electron and hole densities, and set the chemical potential μ equal in the two bilayer sheets. The coupled multiband mean field equations for the gap parameters and the chemical potential at zero temperature are obtained using the standard approach, as described in Ref. [2].

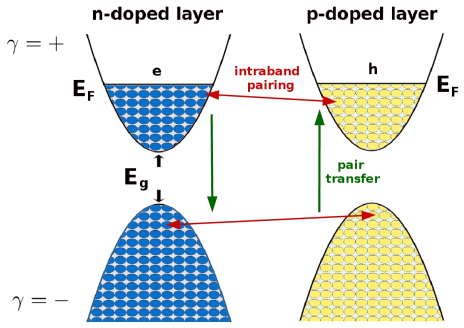

Figure S1. Graphene bilayer electron and hole conduction and valence bands, $\gamma = \pm 1$ after the standard transformation.

[S1] K. Zou, X. Hong, and J. Zhu, Phys. Rev. B **84**, 085408 (2011).
[S2] Y. E. Lozovik and A. Sokolik, Phys. Lett. A **374**, 326 (2009).