Model Updating of Cultural Heritage Buildings Through Swarm Intelligence Algorithms

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Masonry buildings constitute a great Italian historical and cultural heritage, but they were also severely damaged by earthquakes over the centuries. Therefore, to assess their structural and seismic performance, it is crucial to gain a refined and trustworthy numerical model adopting model updating techniques, sometimes based on artificial intelligence algorithms. This paper deals with the development and the updating of a finite element model of an historical church that account for the presence of both the seismic damage and securing systems. The model updating is performed adopting the particle swarm optimization algorithm and is based on the comparison between numerical and experimental modal parameters, the latter achieved by an extensive dynamic test campaign. The obtained calibrated numerical model has been adopted to support the restoration work design, as well as the design of a structural health monitoring system that has been permanently installed on the church.

Keywords: Cultural Heritage Building; Historical Masonry Church; Machine Learning Algorithm; Particle Swarm Optimization; Model Updating; Ambient Vibration Tests.

Introduction

Masonry structures built during the Middle Age and the Renaissance in Italy (e.g., churches, towers, and palaces) constitute an important part of Italian cultural heritage due to their historical value, ongoing community usage, and a large quantity and significance of artworks housed therein. Almost the totality of these structures has been built using unreinforced masonry and in accordance with outdated building regulations, or, for the oldest sites, without restrictions but based on the masters' expertise. Hence, cultural heritage monuments have a prominent vulnerability towards seismic actions, which have been responsible for large damages, leading often to partial or total collapses. Indeed, the numerous earthquakes that occurred in Italy in the past produced many damages to historical buildings, as widely described in the scientific and technical literature, as well as in the historical chronicles and documents. Considering only the last two decades, four strong earthquake sequences hit the Italian country: the 2002 Molise [1], the 2009 L'Aquila [2-4], the 2012 Emilia [5,6] and the 2016 Central Italy earthquakes [7-10]. The latter has been one of the most destructive, producing damages to both historical and common buildings (also newly built) in a wide area of the Central Italy inland. One of the most damaged structural typology was that of churches, which owns intrinsic vulnerabilities, such as the presence of high slender walls, plan and height irregularities and, very often, the lack of box-like behavior. Furthermore, the non-regularity triggered by annex buildings, axisymmetric boundary conditions, and the heterogeneity intrinsic characteristics, generate not conventional failure mechanisms that can be also difficult to predict.

Due to the high occurrence of seismic events and the high number of damaged buildings, the study of the structural performance of heritage masonry constructions has become a priority in the light of preserve the architectural heritage, as well as the human life, being these buildings still used nowadays. A comprehensive work discussing possible structural analysis methods for masonry historical constructions is that of Roca et al. [11], which provides indications about challenging issues on historical structures and strategies for their investigation, considering limit analysis, Finite Element Modelling (FEM) and discrete element methods. These methods have been applied by many authors in the seismic assessment of churches [12-19]. However, realistic models for predicting the expected response are necessary for the analysis of these buildings under severe loads, especially earthquakes.

To obtain a reliable model of a structure, the model updating technique should be used, which is a model calibration procedure to improve the numerical model accuracy and consequently, to achieve trustworthy numerical results [20]. This methodology is based on the idea of reducing the errors between the in-situ outcomes and the numerical results. So, the numerical model can be considered calibrated (or updated) once it represents the behavior of the real structure. A possibility to achieve experimental outcomes from a real building is that of using the Ambient Vibration Test (AVT) methodology, through which the modal parameters of the real structure can be identified adopting the Operational Modal Analysis (OMA) technique. Hence, the most adopted model updating procedure considers the comparison among experimental and numerical modal parameters up to a fair degree of convergence. Model updating techniques are mainly classified into the direct and indirect methodologies [21,22]. The former consists in replicating data obtained from the on-site structure by varying the stiffness and mass matrices; the latter involves adjusting the physical parameters of the model until the observed data are accurately reproduced, and the difference between experimental and numerical results is minimized to an acceptable level. The variation of parameters is usually done manually (manual tuning) at first, performing an initial model optimization changing the main model parameters and determining those that mostly reduce the gap with the experimental results. Then, if results are not yet satisfactory, or the procedure is time consuming, Artificial Intelligence (AI) techniques can be adopted. Indeed, AI methods support the model updating procedure (reducing the time required for the analyses and for their control by the user) and increase the computational efficiency.

Many of the available AI methodologies (e.g., Machine Learning (ML), deep learning, pattern recognition) have been used in the field of structural engineering in the past ten years [23]. Several ML algorithms, such as neural networks, genetic programming, fuzzy computing, and support vector machines are used to predict the mechanical characteristics of the structural (and non-structural) construction materials in buildings and infrastructures [24]. Falcone et al. [25] propose a classification of ML techniques based on the common problems they can solve: neural network is mainly adopted in learning and recognition problems, such as classification, regression and clustering problems; fuzzy computing is mainly employed in fuzzy inference problems, for instance related to uncertainty in input shaping, risk assessment, and control system development. Evolutionary computing (which includes genetic algorithms) and swarm intelligence are mainly used for optimization problems, such as multi-modal and multi-objective optimization tasks.

A comprehensive work that collects methodologies concerning the use of AI in structural engineering in the recent past is that of Salehi & Burgueño [24]. Some examples are reported in the literature discussing the model updating of historic masonry structures. It is the case of Ivorra et al. [26], Casciati & Al-Saleh [27], Gentile & Saisi [28], and Garcia-Macias et al. [29], which described the model updating of historic masonry towers or belfries; or Grosman et al. [30] and Tubaldi et al. [31], which discussed the model updating of masonry bridges; or again Cattari et al. [32,33] and Kita et al. [34] that illustrate the FEM updating of masonry buildings. As concerns the model updating of churches, some examples are available in the literature. Boscato & Cecchi [35], Kujawa et al. [36], Formisano et al. [37] and Di Lorenzo et al. [38] performed the FEM updating of churches adopting the manual tuning technique. Other authors developed automatic algorithms to perform the model updating of churches: Baggio et al. [39], Torres et al. [40] and Sanchez-Aparicio et al. [41] implemented the Douglas-Reid method [42], Boscato et al [43] used an optimization algorithm developed by Adeli and Cheng [44], while Pau & Vestroni [45] implemented an automatic procedure based on the minimization of an objective function. Although a fair number of works dealing with the model updating of historic masonry churches have been produced in the last decade, this topic is far from being considered concluded. Indeed, all the aforementioned works deal with model updating procedures applied to specific case studies, which sensibly differ from each other in geometry, size, main constructive elements, material typologies, etc. Furthermore, there are many model updating procedures available in the literature and applicable for historical buildings and churches, but none of them is nowadays recognized to be better than the others overall. A further increase in number of works dealing with this topic may help to solve these challenges and can support a standardization in the model updating procedures which can be adopted for all types of churches, or at least on church classes with homogeneous characteristics.

This paper deals with the model updating of heritage masonry churches adopting swarm intelligence algorithms. The proposed work is conducted considering the Santa Maria in Via church as case study, which is an old masonry church located in Camerino (Central Italy). This case study can be considered of particular interest because of its high historical and artistic values, and because, after being severely damaged by the 2016 Central Italy earthquakes, the church underwent extensive and massive securing works. So, the model updating has been performed taking into account both the seismic damage and the securing systems built to preserve the structure. A broad description of the church, of its construction materials and techniques, and of the seismic damage, are proposed; then, the extensive AVT campaign is described together with the experimental modal parameters obtained through the OMA technique. After that, a refined FEM of the historical church (also including the securing system) is developed and updated using a swarm intelligent algorithm called Particle Swarm Optimization (PSO). Details of the algorithm and of the performed updating procedure are provided, and comparisons between numerical and experimental results are discussed in order to prove the reliability and usefulness of the proposed approach in calibrating numerical models. Considering the complex nature of the case study (due to the geometry, the construction materials and the presence of damage and securing systems), if the procedure and the model calibration algorithm prove successful, they could be considered as a good practice to be applied in other similar cases.

The Santa Maria in Via church

History overview and description

The Santa Maria in Via church (Figure 1) is situated in the historical town center of Camerino, in the Central Italy Apennines mountains. The original church, built in the 13th century, was smaller and very different than nowadays; later, in the 17th century, the church was merged with the surrounding buildings to form a unique body characterized by a baroque style and with a façade of monumental appearance (Figure 1a). In 1799 a strong earthquake hit the town destroying many buildings; Santa Maria in Via church underwent impressive damage that consisted mainly in the failure of the elliptical masonry dome, which was replaced with a fake dome richly decorated with gypsum stucco and frescoes. Then, in the actual configuration, the church underwent two other important earthquakes in 1873 and 1997, which damaged it again.

The Santa Maria in Via church has a trapezoidal plan shape (Figure 1b) that incorporates a central elliptical hall and four radial chapels, characterized by hemicycle niches, an octagonal sacristy and other rooms. The major axis of the hall passes through the entrance and the opposite main altar located in a deep presbytery; on the minor axis, above two side entrances, there are two small chancels. The plan of the church appears to be massive and strong. Above the main hall, an octagonal tiburium, which approximates the elliptical shape of the hall, rises for more than 8 m with four large windows positioned above the lateral chapels. At the outside, the tiburium is stiffened by buttresses in correspondence of corners. Contained by the tiburium there is a plaster and reed lath dome (commonly known as fake dome) decorated with frescoes (Figure 1c). The dome-drum system is topped by a wooden roof supported by both wooden and steel trusses. On the corner nearby the sacristy there is the bell tower served by a spiral staircase with a rather slender belfry. The façade body has trapezoidal plan shape with an average length of the bases of about 16 m (façade width) and depth of about 6 m (Figure 1b); at the lower part it is connected with the main body of the church, whilst at the upper part with the tiburium, and in the higher part it exceeds the church roof. The interior part of the façade is divided into three levels: the first floor is at 4.5 m from the ground floor and it hosts the organ and the choir, the second floor is located at the level of the first eaves, and the third floor at the level of the clock that characterizes the façade elevation. The façade plan is divided into three rooms, separated by two masonry orthogonal spine walls with communication openings. To access to the choir and to the second floor, a spiral staircase is incorporated into the masonry, producing a significant discontinuity within the wall body.

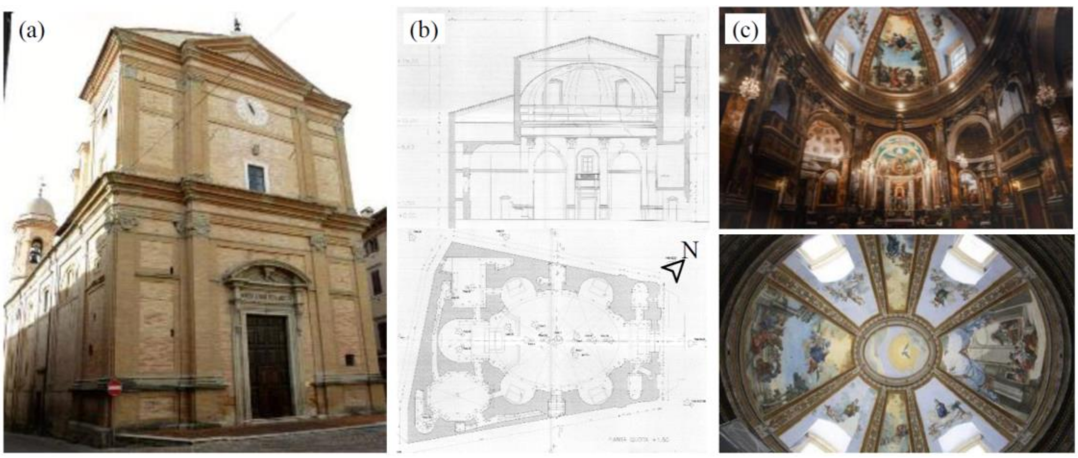


Figure 1. Santa Maria in Via church: (a) exterior picture, (b) architectural drawings, (c) interior pictures.

Damages after the 2016 Central Italy earthquakes

All the historical center of Camerino town was impressively damaged by the 2016 Central Italy seismic sequence. Also, the Santa Maria in Via church suffered many damages, starting from the first shocks of August 24th to the last occurred on January 18th, 2017 (the maximum earthquake magnitude of 6.5 Mw was recorded in Norcia on 30th October 2016, less than 20 Km far from the town). As can be seen from Figure 2, the main damages refer to the failure of the tiburium (mainly the rear part) and, consequently, of part of the wooden roof and fake dome. In addition, the bell tower crumbled on the surrounding houses (fortunately without producing victims), and the façade moved from its vertical position, starting an overturning mechanism. The latter produced an important crack pattern and brick collapses between the façade and the church body. On the contrary, the strong base of the church overcame the earthquakes without significant damages.

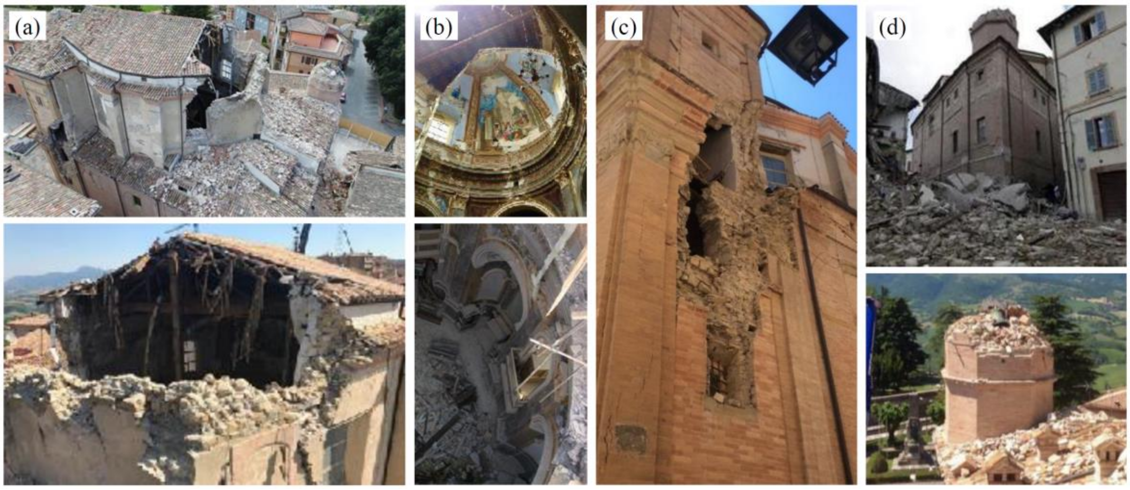


Figure 2. Earthquake damage: (a) collapse of the tiburium, (b) collapse of the dome and interior damages, (c) façade overturning mechanism, (d) collapse of the bell tower.

One of the most impressive failure mechanisms is that activated between the façade and the tiburium: the upper part of the façade is now almost 30 cm out of the vertical plane traced passing towards the low level of the walls, as it has been seen from laser scanner results obtained by surveys after the earthquakes [46]. This damage mechanism also interested the front part of the tiburium, which suffered of diagonal cracks. The horizontal movement of the upper part of the façade explains the formation of shear crack patterns and of the subsequent masonry disaggregation also at the façade base. Finally, the church underwent other minor damages due to the bad weather conditions (heavy snowfalls) occurred during the 2017 winter season

Securing system description

After the seismic sequence that struck the church and produced many damages, the structure has been secured with several interventions. At first, a steel retaining structure was externally built to prevent the façade from the out of plane collapse (Figure 3a). This system has to retain a mass of about 1300 tons with an out-of-verticality of about 30 cm. The structure covers the external sides of the façade (the front side and the two lateral ones), for the whole height. This structure is linked to the main body of the church through fourteen steel strand cables that surround the whole church, with the aim of ensuring the structural stability in case of earthquakes and to confine the base of the church as well. The whole steel structure was founded on a stiff RC shallow foundation. A typical buttress system was not suitable for this case, since it would require a large space in front of the church that would drastically reduce the space necessary to the vehicles mobility and for future reconstruction worksites. Within the holes on the masonry walls produced by the overturning mechanism and by local collapses, steel latticed systems were built with the target of re-establishing the gravity loading pathways.

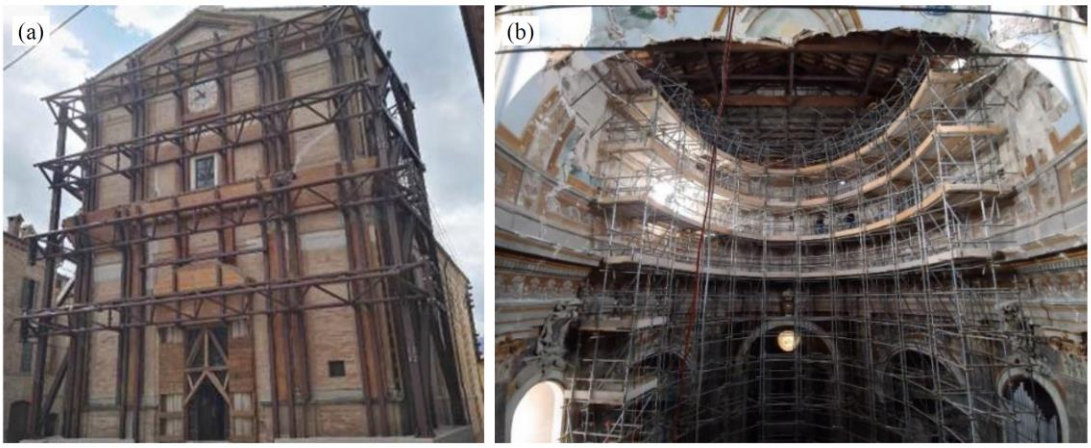


Figure 3. Securing system: (a) exterior structure against façade overturning, (b) interior structure to prevent the collapse of tiburium walls.

Finally, the interior of the church was protected by the construction of a temporary steel roof trusses, and an inner latticed structure (Figure 3b) was also erected to prevent the out of plane collapse of the tiburium walls.

Geometric and construction material surveys

After the securing systems completion, in-situ surveys have been done to periodically control the church health conditions and to perform AVTs. The latter tests have been also adopted to support the design and development of a Structural Health Monitoring (SHM) system.

The in-situ surveys revealed very useful to control the accuracy of the geometric information already available; moreover, they were fundamental to collect information about the morphology of the walls and about the structural elements. Information about geometry and material properties are of paramount importance when the modelling of historic stone/masonry structures is approached, as illustrated in the next Sections. The geometry of the church has been controlled, and the technical drawings are adjourned considering the damages that the church has suffered. The church is characterized by a very complex interior layout, composed by many different volumes with different shapes spread throughout the inner part of the walls and that realize empty spaces at different locations and elevations (Figure 4).

As concerns the construction materials, the church presents a regular external texture realized with brickwork covering, built for aesthetic reasons because of the many variations that it has undergone over the years. Tests for the mechanical characterization of the masonry have not been carried out, but careful visual inspections allowed the individualization of four different masonry typologies, named M1 to M4 (Figure 5a). Masonry M1 consists in two external masonry brick leaves with regular texture, not mutually connected and with rubble infill. This masonry typology was used for the façade perimetric walls. Masonry M2 is a stone masonry with multiple leaves and with irregular courses; this masonry was adopted for the façade inner walls and for some walls of the church body with high thicknesses. Masonry M3 is a doble-leaf brick masonry with rubble infill and, similarly to M1, there is no connection among the several brick layers. This masonry was used to build the greatest part of the church walls, adopting widespread thicknesses and, consequently, different rubble infill thicknesses. Masonry M4 is composed by two leaves: the interior one is made of masonry bricks, while the exterior one of roughly stone blocks with irregular courses, plastered on the exterior face.

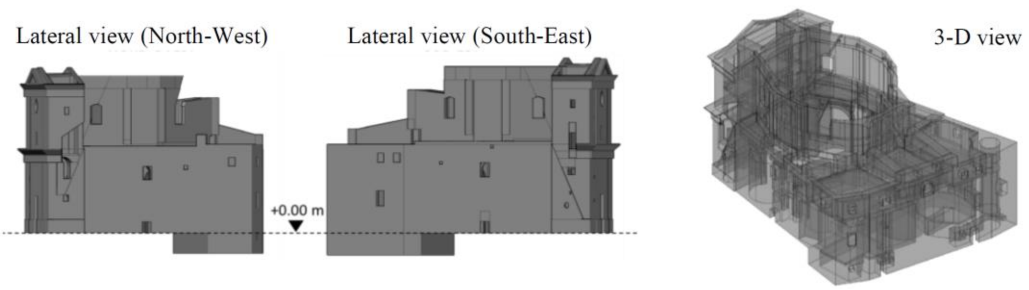


Figure 4. Exterior views and 3D representation of the church geometrical model.

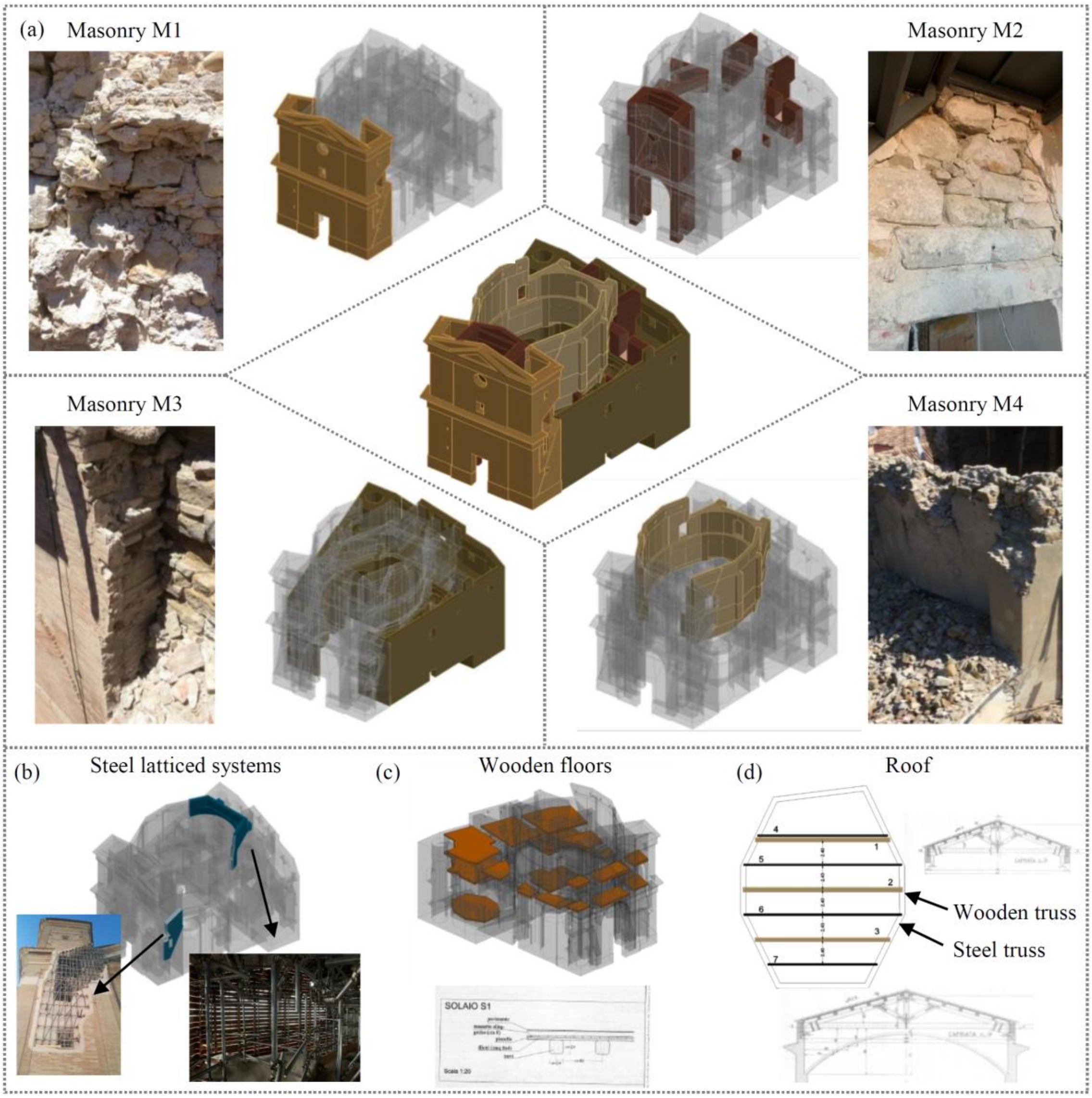


Figure 5. Materials and structural information from in-situ surveys: (a) masonry typologies, (b) steel lattice systems to restore the gravity load paths, (c) interior wooden floors, (d) roof details.

The latter typology was used for the walls of the tiburium. Thus, the church is constituted by a very particular structural system characterized by different interacting bodies sometimes made of poor masonry. This gives the system a very high vulnerability already exhibited after the strong earthquakes occurred in the past.

The rear part of the tiburium and part of the façade lateral sides (that collapsed during the earthquakes) have been reconstructed with steel latticed systems (Figure 5b) that are considered as a new material to be added to those previously described. Within the lateral volumes of the church and at different heights there are many wooden floors (Figure 5c). The roof is composed by three wooden trusses plus four steel trussed (Figure 5d), and by a wooden planking with tile covering

Dynamic characterization through ambient vibration tests

After the completion of the securing works, in the autumn of 2020, an experimental in-situ campaign (consisting in some AVTs) was performed in order to identify the real dynamic behavior of the church. Vibration measurements were performed adopting sixteen mono-axial low-noise piezoelectric accelerometers (PCB model 393B31), connected by means of coaxial cables to NI 9234 analogue-to-digital conversion modules mounted on a cRIO 9045 and on cDAQs 9185 acquisition units (Figure 6b). The acquisition units were connected together by means of ethernet cables, forming a distributed LAN sensor network. The LAN was synchronized using the Time Sensitive Networking (TSN) technology. Also, a laptop equipped with a customized software was used to store data and to control the measurement procedures.

The accelerometers were placed in the interior part of the tiburium and of the façade perimetric walls. Two sensor configurations were adopted, for a totality of twenty-eight single-axis measurement points (Figure 6a); two different height levels were measured, one at the tiburium basis (12 m from the ground) and one at the tiburium mid-height (19.7 m from the ground). This high number of measurement points is rather unusual to find in works dealing with dynamic tests on churches, due to logistical difficulties in positioning sensors in the upper parts of the structure, as well as to limitations relevant to the cultural value of these buildings. Nevertheless, the high number of measurement points revealed to be very helpful for obtaining a very accurate and trustworthy updated FEM of the church at hand, as will be shown in the sequel. The sensors were not positioned at the church lower levels because of the massive wall dimensions, which led to consider this part much more rigid than the upper part. This was also supported by the fact that the lower part of the church did not suffered significant damages after the seismic sequence that produced the massive damage. For each dynamic test, thirty minutes-long acceleration recordings were performed using a 2560 Hz frequency sampling.

The dynamic identification was performed adopting the well-known Principal Component - Stochastic Subspace Identification (SSI-PC) technique [47] and the mode selection was supported by the use of an agglomerative hierarchical clustering algorithm [48] working through the calculation of a distance based on frequency and Modal Assurance Criterion (MAC) [49]. The stabilization diagram obtained from the SSI-PC technique is depicted in Figure 6c, together with the frequency-damping ratio graph; by analyzing them it is possible to see that ten stable vibration modes were clearly identified. The identified modal parameters (frequencies, damping ratios and relevant mode shapes) are summarized in Figure 7.

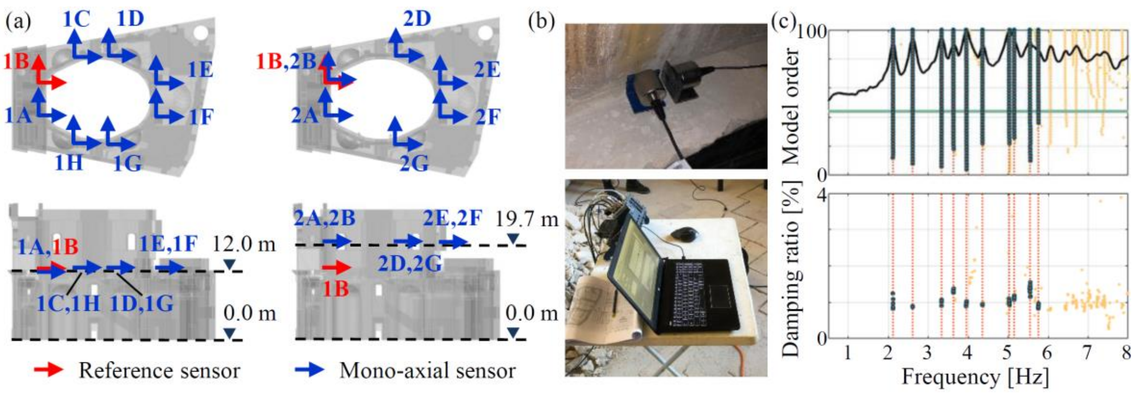
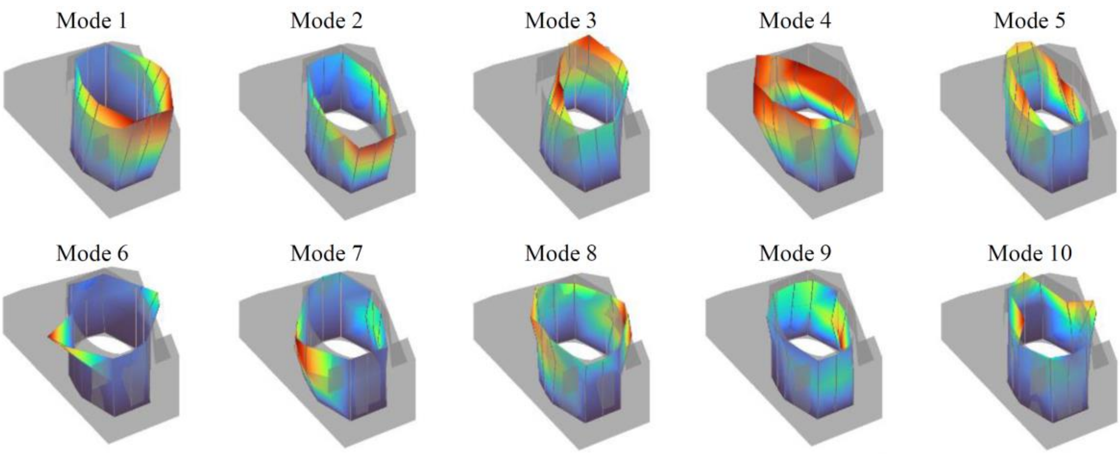


Figure 6. AVTs: (a) sensors layout, (b) adopted instrumentation, (c) stabilization diagram and frequency-damping ratio graph.



Mode	Frequency [Hz]	Damping [%]
1	2.11	0.95
2	2.61	1.01
3	3.34	0.95
4	3.64	1.17
5	3.97	0.93
6	4.37	1.16
7	5.02	1.11
8	5.17	1.06
9	5.56	1.44
10	5.76	1.03

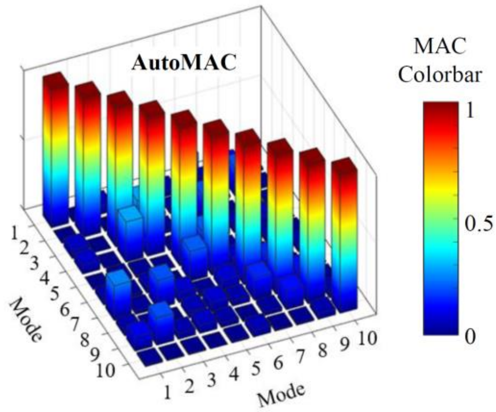


Figure 7. Experimental modal parameters and AutoMAC matrix.

Because of the adoption of two non-simultaneous AVTs, the global mode shapes are obtained adopting the Post Separate Estimation Re-scaling (PoSER) technique [50], scaling the mode displacements by the two reference sensors. All modes are rather global, namely the modal displacements interest almost the totality of the structure; the first and second modes are the first two translational modes in the main orthogonal directions, while the third mode represents the first torsional mode. Although severely damaged, the structure presents very well decoupled modes, also demonstrated by the AutoMAC matrix of Figure 7; it is worth observing that the low level of entropy that characterizes the measurements is similar to that expected from undamaged systems.

Finite element modelling and model updating

Description of the finite element model

The FEM of the church is developed adopting the ANSYS software (Figure 8a); a 3D solid model is created so that the real geometry and thickness of all structural elements, especially the walls, are considered. In detail, solid elements (i.e., SOLID 186 and 187) are adopted to model the whole structure. Furthermore, although part of the damage is simulated by changes in the material mechanical properties (in particular the stiffness, varying the elastic moduli of masonry as will be shown in the sequel), the widespread damage in the tiburium and in the façade is incorporated directly in the geometric model, eliminating those parts that are collapsed and adopting a different material for the masonry that was filled with steel latticed systems. Hence, the numerical model is constructed adopting five different materials: four for the masonry typologies recognized during the in-situ surveys and one representing the parts that are filled with steel latticed systems. The wooden floors within the church are modelled with shell elements (isotropic plates) because they contribute to the development of the box-like behavior of the building, while the roof is not modelled and only its mass is taken into account, because the main part of it collapsed after the earthquakes. Also, the external securing system is modelled since it is in contact with the church façade and, hence, it can affect the whole structural response: the steel trusses and cables that surround the entire church are modelled with beam elements. For what concerns the boundary conditions, the base elements are fixed to the ground. The model is automatically meshed by the software in tetrahedral elements of about 0.5 m of size, achieving a FEM constituted by 538,275 nodes and 309,541 tetrahedral elements (Figure 8b).

All the construction materials are assumed to be homogeneous, elastic, and isotropic. This assumption is considered valid to perform the linear analyses as a support for the design of the restoration works, as well as the dynamic analyses for supporting the SHM system design. The initial values of the main mechanical material properties (elastic modulus, density and Poisson's coefficient) are estimated based on recommendations found in the technical literature and they are listed in Table 1. For the steel elements the canonical steel properties are used, except for the steel latticed systems. This is because the latter are modelled supposing them as solid elements that completely fill the region within masonry that were subjected to collapses, while all the other steel members are modelled with frame elements considering their actual geometric footprint.

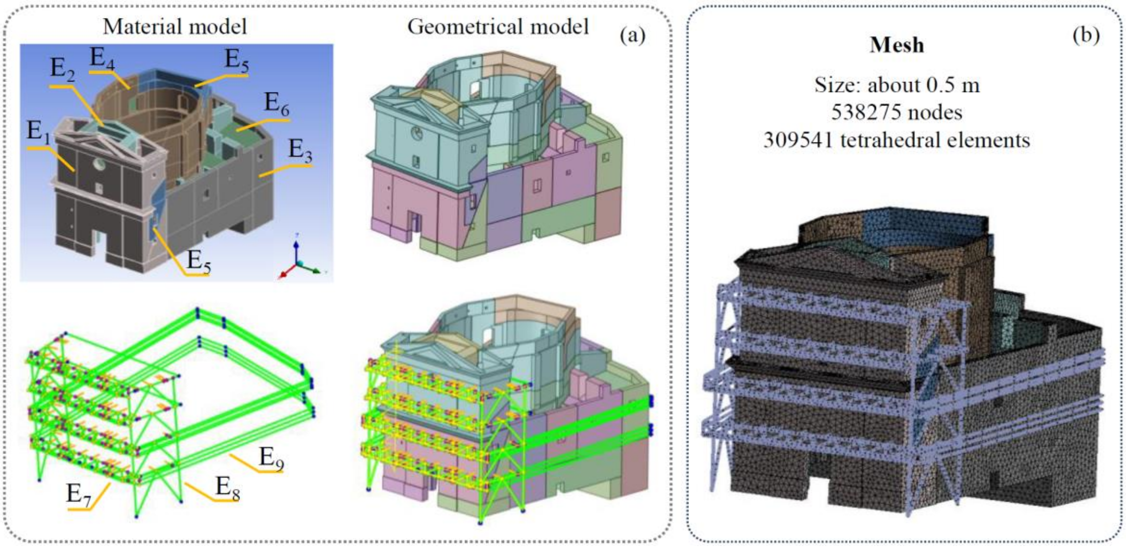


Figure 8. Pictures of the developed FEM and details of the adopted mesh.

Table 1. Initial values for the material mechanical properties.

Materials	Young Moduli E [MPa]	Density [kg/m3]	Poisson coefficient [-]
Masonry M1	1,092 (E1)	1,930	025
Masonry M2	1,230 (E2)	1,930	0.25
Masonry M3	1,200 (E3)	1,930	0.25
Masonry M4	1,365 (E4)	1,930	0.25
Steel latticed systems	1,500 (E5)	1,000	0.25
Wooden floors	600 (E6)	800	0.25
Lattice beams	210,000 (E7)	7,850	0.30
Bracings	210,000 (E8)	7,850	0.30
Strands	110,000 (E9)	7,850	0.30

The particle swarm optimization algorithm

In this work the PSO algorithm [51] is used to calibrate the FEM of the church. This algorithm is a population-based stochastic global optimization technique that belongs to the swarm intelligence family. The basic principle is based on a population of members (called "particles") who evolves via interaction with one another and its surroundings. This algorithm simulates the social behavior of animals (e.g., birds, insects, herds, fishes, etc.). PSO owns several advantages: in addition to the relative simplicity, the fast convergence rate, and the limited number of parameters to be adjusted, it does not require differential, derivative, and continuous optimized function. However, its use must be done consciously because, for functions with several local minima and maxima, it can fall into a local extreme without reaching the true result. PSO is a type of swarm-based search method where each individual (particle) is considered a potential solution to the issue being optimized in the multi-dimensional search space. The individual can remember its best position and that relevant to the swarm; the same occurs for the velocity. During each step, the information of each particle is combined to correct the velocity; this is used to calculate the new particle location. In the multidimensional search space, particles continually alter their states until they achieve equilibrium (or the calculation limits are reached). Unique connection through the different dimensions of the problem is introduced by the objective functions. A flowchart that explains how the PSO algorithm works is shown in Figure 9. An extensive review of successful applications of this algorithm is available in the work of Poli et al. [52].

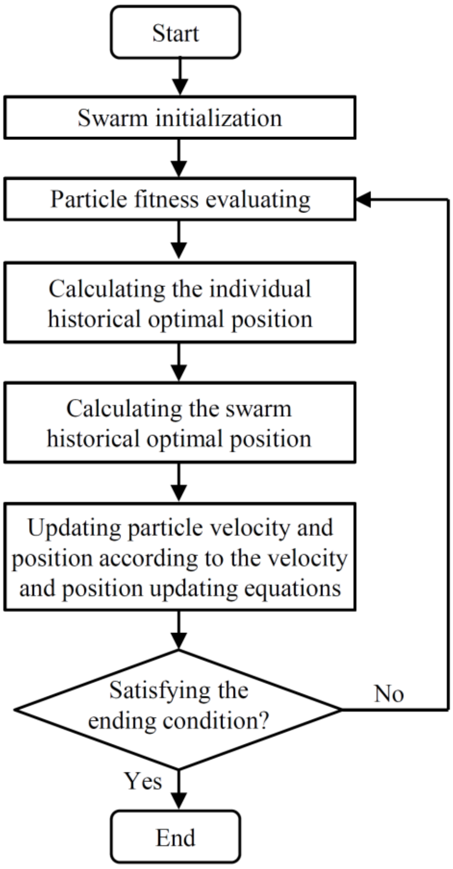


Figure 9. Flowchart of the PSO algorithm.

Mathematically, the PSO can be described as follows; the position vector of each particle in D-dimensional space is $X_i = (x_{i1}, x_{i2}, \ldots, x_{iD})$, the velocity vector is $V_i = (v_{i1}, v_{i2}, \ldots, v_{iD})$, optimal position of each individual (i.e., the optimal position that the particle has experienced) is $P_i = (p_{i1}, p_{i2}, \ldots, p_{iD})$, the optimal position of the swarm (i.e., the optimal position that any individual in the swarm has experienced) is represented as $P_g = (p_{g1}, p_{g2}, \ldots, p_{gD})$. The formulae to update the optimal position of the individual are:

$$p_{i,t+1}^{d} = \begin{cases} x_{i,t+1}^{d} & f(X_{i,t+1}) < f(P_{i,t}) \\ p_{i,t}^{d} & \text{otherwise} \end{cases} \qquad (1)$$

Updating formulas for the velocity and for the position can be written as follows (canonical PSO algorithm [53]):

$$v_{i,t+1}^d = \omega \cdot v_{i,t}^d + c_1 \cdot rand \cdot (p_{i,t}^d - x_{i,t}^d) + c_2 \cdot rand \cdot (p_{g,t}^d - x_{i,t}^d) \tag{2}$$

$$x_{i,t+1}^d = x_{i,t}^d + v_{i,t+1}^d \tag{3}$$

where *rand* denotes a random number in [0,1], ω is the inertia weight, c_1 the cognitive learning factor and c_2 the social learning factor. Analysing the velocity updating formula (Equation 2) from a sociological point of view, it is noteworthy that the first part represents the preceding velocity of the particle, meaning that the particle moves inertially in accordance with its own velocity since it is confident in its existing state of motion. The second part is inherent to the distance between the current position of the particle and its optimal position, and this is called the "cognitive" item. The third part depend on the distance relating the current position of the particle and the optimal position of the swarm, named "social" factor. Generally, inertia weight ω is employed to find a balance between local and global searches: high values lead to global search, while low values to local search; therefore, over time, this value should steadily decrease.

Model updating

A model updating procedure is developed in order to obtain a calibrate FEM that reproduces the real behavior of the church. More specifically, the updating has been performed with the target to replicate numerically the real dynamic behavior experimentally identified through AVTs. Generally, several parameters can be considered in the updating procedures, as the material mechanical properties (elastic moduli, Poisson's ratio), the masses, the geometry of the modelled elements, the boundary conditions, the presence of damage, etc. The selection of parameters to be updated is often based on global or local sensitivity analyses that give an indication of those variables that mostly affect the model response. However, in this work a rigorous sensitivity analysis has not been performed, going beyond the scope of the paper and requiring a very high computational effort due to the FEM complexity; therefore, only an initial FEM manual tuning has been done to have an indication on which parameters mostly affect the dynamic behavior of the modelled church. After this simple investigation, the elastic moduli of the construction materials (four masonries plus the steel-filled damaged areas – E1 to E5) are assumed as updating parameters. Lower Bound (LB) and Upper Bound (UB) elastic modulus values for the four masonry typologies are chosen based on values reported in Table C8.5.I of the Circular of the Italian Technical Code [54]. Conversely, reasonable LB and UB for the elastic moduli of the steel latticed system are proposed on the basis of the Authors' experience. The intervals of the elastic moduli considered in the updating process are decided in accordance with the initial tentative values proposed in Table 1, and they are listed in Table 2. The masonry masses are excluded from the updating parameters since they can be calculated with rather good accuracy (construction typologies are known); the same can be assumed for the masses of the steel latticed systems and for the wooden floors. The base restraint stiffness is not considered as updating parameter because the dynamic test results demonstrated the very high rigidity of the base with respect to the upper part of the church body. Also, the Poisson's coefficients of masonries are assumed constant because it has been demonstrated [43] that they do not significantly influence the dynamic response of masonry buildings; a value equal to 0.25 is assumed in this work, being included in a reasonable range found in the scientific literature (0.15 – 0.30).

The updating procedure can be considered as an optimization problem in which a set of parameters $x = \{x_1, x_2, \ldots, x_n\}$ can be defined as optimal, and which is able to minimize (or maximize) some system characteristic that is dependent on x. In this work, the model updating process is based on the comparison between the identified experimental (Exp) modal parameters (from AVTs and OMA) and the relevant numerical (Num) ones (derived from the developed FEM).

Table 2. Initial intervals of the updating parameters and values reached at the end of the updating procedure.

Updating parameter	Acronym	Elastic moduli [MPa]		
		Extreme values		Updated value
		LB	UP	
Masonry M1	E1	200	2,200	605
Masonry M2	E2	200	2,200	415
Masonry M3	E3	200	2,200	401
Masonry M4	E4	200	2,200	1,542
Steel latticed systems	E5	100	20,000	16,102

In detail, natural frequencies and mode shapes of the first ten vibration modes are considered in the updating procedure, which ends once the numerical outcomes (modal parameters) fit well the relevant experimental ones. The good matching is evaluated calculating the difference between the numerical and experimental frequencies, as well as taking into account the MAC indexes [55]. Thus, a combined objective function which considers simultaneously differences both in terms of frequencies and mode shapes is proposed and adopted, and it can be written as follows:

$$EF = \ln\left(1 + \left|\frac{f_{num}(\mathbf{E}) - f_{exp}}{f_{exp}}\right| + \left(1 - MAC_{num,exp.}(\mathbf{E})\right)\right) \tag{4}$$

Equation 4 is called Error Function (EF) and the model updating process is aimed to minimize it as much as possible, i.e., to solve the problem:

$$\min_{\boldsymbol{E}} EF \qquad \text{with} \qquad \boldsymbol{E} = \{E1, E2, E3, E4, E5\} \tag{5}$$

The EF minimization problem is solved through the use of the PSO algorithm: in this case, particles are represented by the calculated EF values and a Matlab routine is implemented to automatically adjourn the updating parameters on the basis of the PSO rules and to automatically perform (in ANSYS environment) numerical modal analyses. In detail, the first numerical analysis is performed on the FEM with the initial tentative values of the elastic moduli listed in Table 1. Then, E1 to E5 are iteratively modified (within the limits of Table 2) through the use of the PSO algorithm up to the minimization of the EF of Equation 4. It is noteworthy that a reasonable number of iterations should be performed in each PSO iteration step [51]. The Authors' choice was ten iterations for each parameter, which led to perform fifty elastic modulus iterations with the relevant fifty numerical modal analyses at each PSO iteration step. This means that each fifty modal analyses the algorithm choses the set of parameters that led to the best solution, namely that minimizes the EF, and then re-start from this one to perform numerical analyses. The iterative updating procedure ends when both the EF and the elastic moduli reach values that remain constant as iterations increase.

Figure 10 summarizes the results obtained at the end of the updating, where the graphs of the EF and elastic moduli evolution are reported as a function of the PSO iteration steps. As can be seen, the procedure ended in correspondence of the 43rd PSO iteration, when the EF assumed a constant values of about 1.42, which was the lowest one during all the process. It is worth remembering that fifty numerical analyses were performed for each PSO iteration; so, a totality of 2,150 numerical modal analyses have been performed, for a total duration of the analysis of about 1 week. Obviously, the convergence of the EF to a lower value is not always expected, especially when few and rough data are provided for the calculation of the EF. In this work, a very good result was achieved (EF = 1.42) because of the high number of vibration mode considered in the calculation of the EF, which in turn depends on the high number of accelerometers employed during AVTs.

The elastic moduli obtained at the end of the updating procedure are listed in Table 2. Through the comparison between those assumed as initial ones (Table 1), some considerations can be drawn.

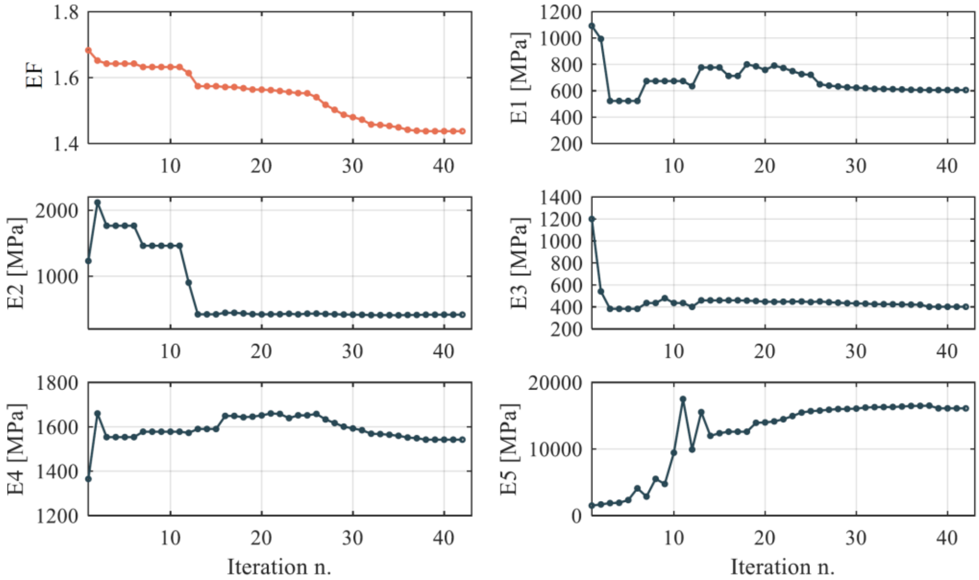


Figure 10. EF and elastic moduli trends during the updating procedure performed with the PSO algorithm.

As expected, it is evident that the majority of masonries reduce their stiffness due to the presence of seismic damage. In detail, the updated elastic moduli of M1 and M2 are much lower than the initial ones and this is in accordance with the severe and widespread crack patterns detected on the walls made with this masonry typologies. Also, masonry M3 has a value much lower with respect to the beginning of the procedure but, contrarily from M1 and M2, the walls of the lower part of the church (M3) were not severely damaged by the earthquakes. However, this very low value can be justified by the presence of slight damage and by the fact that the base walls, having very high thicknesses, are surely made with greater rubble infills that lead to a lower stiffness of the whole walls. Contrarily than before, masonry M4 has a final elastic modulus value that slightly increase, and this may be explained by the presence of the inner latticed structure that was built to avoid the collapse of the tiburium walls. The elastic modulus of the steel lattice system volumes is sensibly higher at the end, but, as stated before, the initial tentative value was roughly estimated by the Authors since no evidences have been found in the technical literature.

Discussion of results

The comparison between the experimental and numerical dynamic behavior of the church before and after the model updating is made to assess the reliability of the elastic moduli obtained at the end of the PSO procedure (Table 2). As can be seen from Figure 11a, the ten numerical vibration modes of the updated model are very similar to those experimentally identified and represented in Figure 7. Indeed, all of them are global modes and the first five mobilize the greatest part of the church body; moreover, the first three modes still represent the first transverse, first longitudinal, and first torsional ones. Comparing the numerical frequencies after the updating with the experimental ones (Table 3) it is possible to observe that they are in good agreement, with percentage differences almost always lower that 10%. In addition, the numerical fundamental mode perfectly matches the relevant experimental one, being the two frequencies almost equal (numerical frequency 2.08 Hz vs experimental frequency 2.11 Hz).

The comparison between mode shapes is also made considering the MAC index, which give a quantitative indication about the modal shape likeliness. The assessment is made comparing each numerical modal shape with all the experimental ones, and MAC results are collected in the MAC matrix of Figure 11b. As can be observed from the diagonal entries of the matrix after the updating, the first five numerical mode shapes are in good accordance with the relevant experimental ones, while the matching is less precise (but still very acceptable) for the other five.

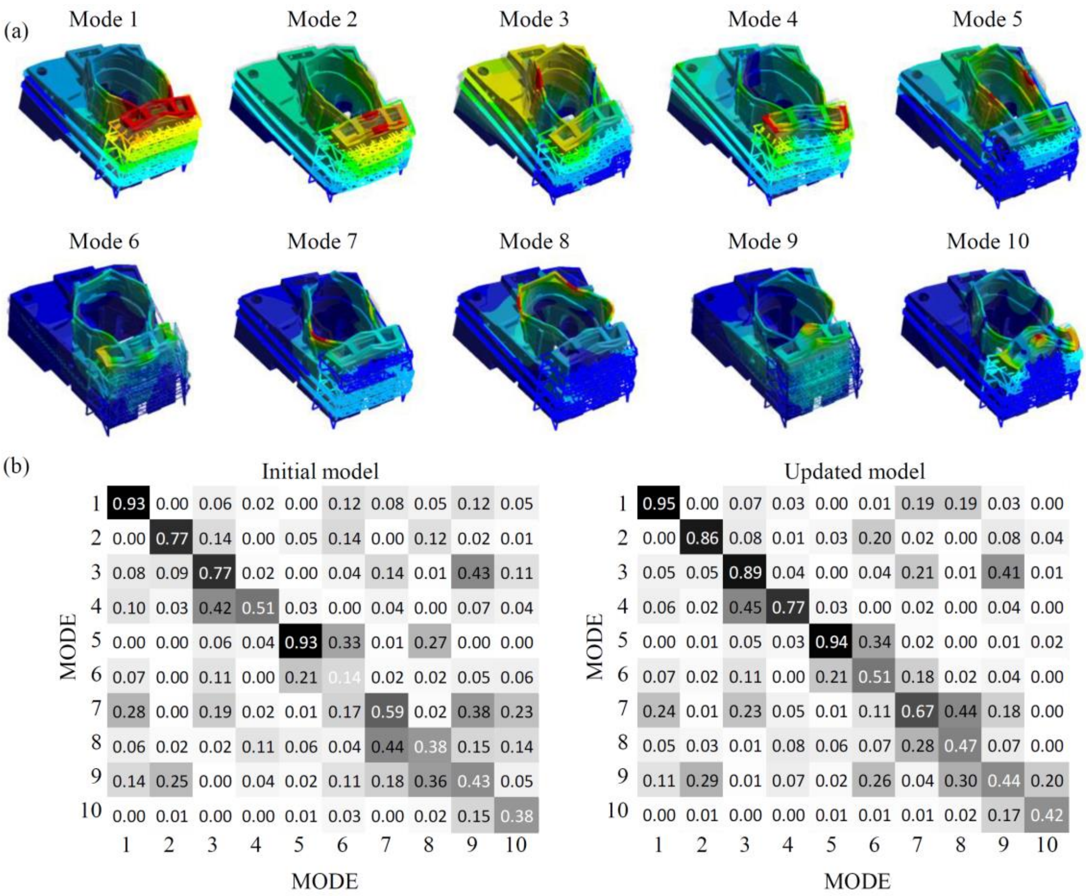


Figure 11. Numerical results: (a) mode shapes of the calibrated FEM, (b) comparison between num. and exp. mode shapes before and after the FEM updating.

Table 3. Comparison between numerical and experimental frequencies before and after the FEM updating.

Mode	Exp. [Hz]	Initial Model		Updated model	
		Num. [Hz]	Δ [%]	Num. [Hz]	Δ [%]
1	2.11	1.95	8	2.08	1
2	2.61	2.58	1	2.76	-6
3	3.34	2.69	20	2.92	13
4	3.64	3.43	6	3.3	9
5	3.97	3.07	23	3.86	3
6	4.37	4.82	-10	4.66	-7
7	5.02	5.99	-19	4.78	5
8	5.17	4.71	9	5.56	-8
9	5.56	5.39	3	5.94	-7
10	5.76	5.32	8	6.28	-9

The elastic moduli estimated adopting the PSO algorithm are much more reliable in reproducing the real dynamic behaviour of the church with respect to the initial ones, as proven by the improvement in the comparison between modal parameters before and after the model updating. This demonstrates that, although the initial FEM was very refined, it was not sufficiently representative of the actual behaviour of the structure because of the importance in the correct estimation of the construction materials elastic moduli. The latter are very difficult to obtain in historical buildings because of the restrictions in performing destructive tests, as well as the non-homogeneity of the masonry throughout the structure. A FEM updating procedure (as that previously shown) could support this task.

Conclusions

The paper dealt with the updating of a finite element model of an historical church located in Central Italy through the use of an artificial intelligence algorithm. The case study is the Santa Maria in Via church located in Camerino town, severely hurt by the 2016 Central Italy earthquakes. The uniqueness of this work lies in the fact that the updated model account for both the seismic damage and the securing system built to preserve the structure.

At the beginning of the paper, a comprehensive description of the church has been provided, together with a detailed explanation of the securing works. Indeed, after being damaged by seismic actions, the church underwent securing works that consisted in many invasive interventions, such as the construction of numerous steel structures to prevent further collapses. The most important one consists in the construction of a steel retaining system that encloses the façade and that is anchored to the building by means of steel cables that surround the entire church. After the completion of these works, an extensive experimental campaign was performed with many purposes: to detect and describe all damages, to control the geometric dimension, to individuate the masonry typologies, and to perform dynamic tests. The latter allowed the identification of the real dynamic behavior of the whole church in its actual state (damaged and secured).

The data collected during the inspections, together with those found on technical drawings, allowed the construction of the finite element model of the church in ANSYS environment. This model was built with solid elements with the particularity of including both the seismic damage and the main securing systems. Then, the finite element model has been updated: the elastic moduli of the construction materials (mainly masonry) were selected as updating variables since it was found that they mostly affected the numerical behavior of the model. It is worth highlighting that a manual tuning sensitivity analysis was perform before deciding the parameters to be updated. For the model updating, an artificial intelligence algorithm was adopted; in detail, the particle swarm optimization algorithm, which belongs to the swarm intelligence family of the machine learning methods. The algorithm was used implementing a Matlab routine that allowed for the automatic execution of analyses, and at the end of the procedure the elastic moduli of the considered materials have been obtained. The iterative updating procedure has reached convergence with a minimum level of error also by the high number of vibration modes considered for the calculation of the error function, which in turn depends on the high quality of the performed in-situ dynamic tests. The reliability of the elastic moduli obtained at the end of the procedure was proven comparing the modal parameters of the calibrated finite element model (namely, the first ten vibration mode frequencies and mode shapes) with the relevant ones experimentally identified by performing the in-situ dynamic tests. The comparison showed a very good matching.

The proposed procedure and updating algorithm revealed to be effective in achieving the target of the research, namely to obtain a calibrated model of the church case study; moreover, in this work, the model updating is performed considering both the seismic damage and the presence of the securing systems, and this is a very uncommon fact that has not been found in the literature so far. Consequently, the proposed approach could be applied in other similar case studies to further assess its effectiveness, which, if proven, could lead to the development of a good practice for the model calibration of historical masonry buildings, both in healthy state and damaged.

The updated model of the church can be used for many purposes, along with the execution of the analyses required for the assessment of the building safety conditions, as well as the design of restoration works. Furthermore, a deep knowledge of modal characteristics and the availability of an updated model is the first step in a structural monitoring program. Indeed, the updated model was used to support the design of a monitoring system that has been permanently installed on the church and that could reveal fundamental to individuate any decay over time in the structural integrity of the monument. The latter research is still ongoing and the main results of the monitoring will be disseminated in the next future.

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