



**UNIVERSITÀ DEGLI STUDI DI CAMERINO**  
**School of Advanced Studies**

**DOCTORAL COURSE IN**  
***COMPUTER SCIENCE AND MATHEMATICS***  
**XXXVII cycle**

**Reliability Assessment of Existing Concrete  
Bridges: Knowledge Uncertainties,  
Degradation Phenomena, and Seismic Risk**

**PhD Student**  
**Alberto Poeta**

**Supervisor**  
**Prof. Andrea Dall'Asta**

**Co-supervisor**  
**Laura Gioiella**  
**Co-supervisor**  
**Fabio Micozzi**

Part of the research (Chapters 6 and 7) has been developed within the program of the project SECURE - Scuola delle Ricostruzione e Resilienza, funded by Agenzia per la Coesione Territoriale (D.283/2021)

The PhD fellowship has been co-funded by Consortium REDI and UniCAM-SAS



## Contents

|          |                                                                          |    |
|----------|--------------------------------------------------------------------------|----|
| <b>1</b> | <b>INTRODUCTION</b>                                                      | 7  |
| 1.1      | REFERENCES                                                               | 11 |
| <b>2</b> | <b>STRUCTURAL RELIABILITY</b>                                            | 12 |
| 2.1      | FAILURE PROBABILITIES AND RELIABILITY INDEX                              | 12 |
| 2.2      | SAFETY FACTOR AND SEMI-PROBABILISTIC APPROACH                            | 16 |
| 2.3      | TIME-DEPENDENT RELIABILITY                                               | 18 |
| 2.4      | RELIABILITY OF STRUCTURAL SYSTEM                                         | 22 |
| 2.5      | ASSESSMENT METHODS                                                       | 24 |
| 2.5.1    | Numerical Integration                                                    | 24 |
| 2.5.2    | Analytical Approximation                                                 | 25 |
| 2.5.3    | Simulation Techniques                                                    | 28 |
| 2.6      | REFERENCES                                                               | 37 |
| <b>3</b> | <b>POST-TENSIONED BRIDGES</b>                                            | 39 |
| 3.1      | INTRODUCTION                                                             | 39 |
| 3.2      | GENERAL PRINCIPLES                                                       | 41 |
| 3.3      | DESIGN ACCORDING TO LIMIT STATE                                          | 46 |
| 3.3.1    | Serviceability Limit State (SLS)                                         | 47 |
| 3.3.2    | Ultimate Limit State (ULS)                                               | 48 |
| 3.4      | BASIC RANDOM VARIABLES                                                   | 50 |
| 3.4.1    | Material Properties                                                      | 51 |
| 3.4.2    | Prestressing Force                                                       | 60 |
| 3.4.3    | Loads                                                                    | 61 |
| 3.5      | REFERENCES                                                               | 65 |
| <b>4</b> | <b>RELIABILITY OF EXISTING POST-TENSIONED CONCRETE BRIDGES</b>           | 67 |
| 4.1      | INTRODUCTION                                                             | 67 |
| 4.2      | SEMI-PROBABILISTIC EVALUATION AND NOTATION                               | 67 |
| 4.3      | EFFECT OF KNOWLEDGE UNCERTAINTIES ON THE ESTIMATION OF DESIGN PARAMETERS | 71 |
| 4.4      | FULLY PROBABILISTIC EVALUATION                                           | 75 |
| 4.5      | REFERENCES                                                               | 77 |

|          |                                                                                                                    |            |
|----------|--------------------------------------------------------------------------------------------------------------------|------------|
| <b>5</b> | <b>CASE STUDY .....</b>                                                                                            | <b>78</b>  |
| 5.1      | INTRODUCTION .....                                                                                                 | 78         |
| 5.2      | DESCRIPTION OF THE CASE STUDY .....                                                                                | 78         |
| 5.3      | SEMI-PROBABILISTIC ANALYSIS AND SAFETY COEFFICIENT<br>80                                                           |            |
| 5.3.1    | Perfect Knowledge Scenario .....                                                                                   | 80         |
| 5.3.2    | Partial Knowledge Scenario without Measurement Errors .....                                                        | 82         |
| 5.3.3    | Partial Knowledge Scenario with Measurement Errors .....                                                           | 83         |
| 5.4      | PARAMETRIC ANALYSIS AND RELATED RESULTS .....                                                                      | 83         |
| 5.4.1    | Outcomes of the Assessment through Tests without Measurement<br>Error                                              | 84         |
| 5.4.2    | Outcomes of the Assessment through Tests with Measurement Error<br>89                                              |            |
| 5.4.3    | Applicability and Limitations of the Outcomes of the Safety<br>Assessment Considering Knowledge Uncertainties..... | 94         |
| 5.5      | PROBABILISTIC ANALYSIS AND FAILURE PROBABILITY .....                                                               | 95         |
| 5.5.1    | Basic Random Variable for Reliability Assessment.....                                                              | 95         |
| 5.5.2    | Probabilistic Analysis Tool.....                                                                                   | 96         |
| 5.5.3    | Methodology .....                                                                                                  | 97         |
| 5.5.4    | Probabilistic Outcomes .....                                                                                       | 97         |
| 5.6      | REFERENCES.....                                                                                                    | 103        |
| <b>6</b> | <b>TIME-DEPENDENT RELIABILITY AND DEGRADATION<br/>PHENOMENA .....</b>                                              | <b>104</b> |
| 6.1      | INTRODUCTION .....                                                                                                 | 104        |
| 6.2      | CORROSION REINFORCING STEEL.....                                                                                   | 104        |
| 6.2.1    | Initiation Time.....                                                                                               | 105        |
| 6.2.2    | Corrosion Rate .....                                                                                               | 108        |
| 6.2.3    | Corrosion Effect on Reinforcement .....                                                                            | 111        |
| 6.2.4    | Comparison Among Methods .....                                                                                     | 118        |
| 6.3      | METHODOLOGIES FOR TIME DEPENDENT SEISMIC RISK<br>ASSESSMENT .....                                                  | 122        |
| 6.3.1    | Seismic Action: Stochastic Ground Motion Models.....                                                               | 122        |
| 6.3.2    | System Capacity.....                                                                                               | 123        |

|           |                                                                                  |            |
|-----------|----------------------------------------------------------------------------------|------------|
| 6.4       | REFERENCES.....                                                                  | 125        |
| <b>7</b>  | <b>CASE STUDY ANALYSIS .....</b>                                                 | <b>128</b> |
| 7.1       | INTRODUCTION .....                                                               | 128        |
| 7.2       | Description of the Case Study.....                                               | 128        |
| 7.3       | Time Evolution of Cross Section Statistical Properties .....                     | 129        |
| 7.4       | Seismic Hazard.....                                                              | 134        |
| 7.5       | Hazard Rates and Failure Probabilities.....                                      | 135        |
| 7.6       | Sensitivity of Random Parameters .....                                           | 138        |
| 7.7       | REFERENCES.....                                                                  | 141        |
| <b>8</b>  | <b>CONCLUSION.....</b>                                                           | <b>142</b> |
| <b>A.</b> | <b>APPENDIX: STATISTICAL CONCEPTS.....</b>                                       | <b>144</b> |
|           | RANDOM VARIABLES .....                                                           | 144        |
|           | JOINTLY DISTRIBUTED RANDOM VARIABLES .....                                       | 146        |
|           | PROBABILISTIC DISTRIBUTIONS USED .....                                           | 148        |
|           | Normal or Gaussian Distribution .....                                            | 148        |
|           | Lognormal Distribution.....                                                      | 148        |
|           | Poisson and Exponential Distribution.....                                        | 149        |
|           | Beta Distribution .....                                                          | 150        |
|           | Gumbel Distribution .....                                                        | 151        |
|           | Weibull Distribution.....                                                        | 152        |
|           | STATISTICAL INFERENCE .....                                                      | 153        |
|           | Estimation of the Samples Mean with Known and Unknown Population<br>Variance 154 |            |
|           | Estimation of the Sample Variance .....                                          | 156        |
|           | POISSON AND RENEWALL PROCESS.....                                                | 158        |
|           | REFERENCES.....                                                                  | 159        |

# 1 INTRODUCTION

The transport network plays a fundamental role in global economic and social development. It is a crucial factor for the economic growth and it performs an essential function in daily lives, allowing the fast, efficient, and safe movement of people and goods. A significant portion of this network consists of roadway infrastructures.

Bridges constitute a significant portion of this asset. According to [1] most bridges in northern and central European countries were constructed between 1946-1965. In Italy, a large part of bridges stock was built between the '60s and '80s decades when the country experienced its economic growth and infrastructure development [2]. As a result, nowadays bridge network primarily consists of aging structures that are still in service and that have been designed by old codes. Moreover, these infrastructures are subjected to loads greater than those considered in the design due to increasing of the total traffic and travelling loads. Furthermore, many bridges have exceeded their design lifetime and some of them show a significant degradation due to environmental effects also caused by the insufficient investments in maintenance interventions. All these aspects and their crucial importance have been highlighted by unexpected collapses of bridges occurred worldwide in the recent past, causing economic and social losses. Among them, the collapse of the Fossano overpass in Cuneo in 2016 (IT), the Polcevera viaduct in Genova in 2019 (IT), the Troja footbridge in Prague in 2017 (RC), the Nanfangao bridge in Taiwan in 2019, and the most recent Fern Hollow Bridge in 2022 in Pittsburgh (US).

From the economical point of view, recent studies have estimated the budget to renovate bridges in different countries. The COST ACTION 345 report [3] estimated a budget of about 400 billion Euros for the replacement costs of one million of existing bridges in the 27 European countries. In 2013 the Federal Highway Administration [4] estimated that approximately the 25 percent of US bridges were either structurally deficient or functionally obsolete. In 2024 the ASCE Infrastructure Report Card [5] estimates that the bridge-related system rehabilitation needs 191 billion dollars.

These challenges underscore the necessity to develop efficient and effective assessment methods to evaluate the structural reliability of existing bridges. Such methods are essential for supporting stakeholders in optimizing the maintenance and restoration interventions and for ensuring the strategical allocation of limited maintenance budgets.

Among all bridge types, the Post-Tensioned (PT) concrete bridges pose a particular challenge due to the high level of uncertainty involved in assessing their vulnerability. Such uncertainty concerns the knowledge of the state of conservation of the precompression system, which cannot directly be investigated through visual inspection or conventional tests. Various countries have addressed this issue by introducing specific guidelines, such as those established in the US [4], England [5]-[7], and more recently in Italy [8].

The evaluation of the structural reliability can be carried out through two main approaches: the semi-probabilistic approach, widely used among practitioners, and the probabilistic approach. In the semi-probabilistic approach, uncertainties are considered through characteristic parameters combined with partial safety factors, calibrated according to target values of the failure probability. The outcome of this approach is a safety ratio, typically expressed as the capacity/demand ratio. The partial factors recommended by most national and international Standards have been developed for the assessment of new structures [9]. However, they can also be applied for existing structures. In the probabilistic approach, instead, uncertainties are explicitly incorporated by considering the probability distributions of the basic random variables. In this case, structural reliability is commonly expressed in terms of the annual failure probability.

For new structures, the adoption of semi-probabilistic reliability assessment results in a deterministic safety ratio that derives from the knowledge of predefined deterministic design parameters, based on statistical properties of the basic variables. In fact, in the case of new structures all the variabilities of the basic variables are considered defined and constrained by the current production and construction standard. Differently, the safety ratio of for existing structures becomes an uncertain quantity since the statistical properties of the basic variables are not known in advance, and they can only be estimated from a limited number of imprecise tests. Similarly, in the probabilistic approach, the failure probabilities of existing structures can only be estimated, since the probability distributions of the basic random variables must be derived from a limited set of tests results. Although the impact of knowledge uncertainties on reliability assessment of existing structures is treated in literature, as underscored in the study reported in [10], no previous studies have examined this effect on PT members. These structures are characterized by a greater complexity, a higher number of variables, and potentially higher uncertainties about measurement errors.

In summary, up to now, the topic of evaluating the safety assessment of existing PT bridges still has some points requiring to insight. Consequently to fill these voids, the first objective of this thesis is to evaluate and to discuss the influence of knowledge uncertainties on the reliability assessment of existing PT beams performing both the semi-probabilistic and the probabilistic approach. Specifically,

the study focuses on the estimation of design parameters (for the semi-probabilistic approach) and on the probability distributions of basic variables (for the probabilistic approach), based on in-situ tests results. The impact of the knowledge parameters on the assessment results are investigated including *i*) the number of tests; *ii*) the confidence level chosen for the estimation of the design parameters; *iii*) the measurement errors. The analysis is carried out through three different knowledge scenarios. In the first scenario, an ideal condition of “perfect knowledge” is assumed. The safety assessment in this scenario serves as the reference solution. In the second scenario, the knowledge process is considered: the influence of the number of tests on the safety assessment is studied, considering also different choices for the confidence level assumed for the estimation of the parameters. In the third scenario, the error affecting the experimental measurement is introduced. The study evaluates the safety of PT beams with reference to failure conditions related to bending moment and shear, which are significantly influenced by the precompression system.

Another essential aspect worth to be investigated for existing structures is the exposure to aggressive environmental conditions. They induce degradation processes, such as corrosion, that reduce the structural performance. Degradation is primarily caused by the diffusion of chemical components, such as carbon dioxide and chlorides. Chlorides are critical issue for huge stock of existing concrete bridges. The corrosion deterioration significant affects PT members capacity under static load at both serviceability and ultimate limit state. However, corrosion effect is even more relevant when structures are subjected to seismic loads, because its process affects not only strength but also material ductility. Therefore, the second objective of this thesis is to provide a comprehensive reliability assessment of existing bridges, by examining the influence of corrosion deterioration within seismic risk that is one of the most critical natural. When degradation is incorporated into seismic risk assessment, the number of basic variables to be considered further increases, leading to greater system complexity and uncertainty. Being structural degradation a process that involves over-time, it is worth nothing that the additional uncertainties influence the strength and the reliability of reinforced concrete bridge piers over time. Consequently, seismic risk is described by the time-dependent annual rate of exceedance of the ratio between seismic demand and strength capacity. Failure rate and reliability over a time interval can be derived by considering the probability of exceedance of the unit value. Compared to previous literature studies, the fundamental purpose of the second section of this work is to identify which parameters most influence the system in order to develop simplified models that can be used in Daily Engineering Evaluation in the future, both from Code and Engineering sides.

To achieve these two goals, the thesis is structured as follows: the second chapter introduces the fundamental concepts of reliability, along with the computational

methods; the third chapter examines PT bridges, discussing their design criteria, fundamental limit states, and basic random variables affecting their structural response; the fourth chapter explores the evaluation of the existing structures adopting both the semi-probabilistic approach and the probabilistic approach; the fifth chapter inspect the impact of knowledge uncertainties on the assessment process and consequently on the safety assessment taking into account a real case study evaluated through three different knowledge scenarios; the sixth chapter describes the phenomena leading the reinforcement corrosion and it examines the combined effect of seismic hazard and structural degradation; finally, the seventh chapter inspects the impact of degradation phenomena on the seismic capacity taking into account the bridge pier of the case study analysed in the fifth chapter.

## 1.1 REFERENCES

- [1] Samaris, 2005. State of the Art Report Assessment of Structures in Selected EEA and CE Countries – Deliverable D19. Sustainable and Advanced Material for Road Infrastructure. V Framework program, Brussels.
- [2] Pinto, P.E.; Franchin, P., 2010. Issues in the Upgrade of Italian Highway Structures. *Journal of Earthquake Engineering*, 14, 1221–1252, doi:10.1080/13632461003649970.
- [3] COST 345, 2004. Procedures for the assessment of Highway Structures. Final Report, Reports of Working Groups 1-6.
- [4] Federal Highway Administration-HRT-13-028: Guidelines for Sampling, Assessing, and Restoring Defective Grout in Prestressed Concrete Bridge Post-Tensioning Ducts. 2013
- [5] U.S. Department of Transportation, Federal Highway Administration, 2024. “Status of the Nation’s Highways, Bridges and Transit: Conditions and Performance,” 25th ed., Report to Congress.
- [6] Highways England, 2020. CS 465 Management of Post-Tensioned Concrete Bridges.
- [7] Highways England, 2020 CS 464 Non-destructive testing on highways structures.
- [8] Italian Ministry of Infrastructures and Transport, 2020. Decree 01/07/2022, n. 204, Guidelines on Risk Classification and Management, Safety Assessment and Monitoring of Existing Bridges, Italian Ministry of Infrastructures and Transport: Rome, Italy.
- [9] CEN EN 1990 Eurocode 0- Basis of Structural Design. CEN 2002, Brussels
- [10] Caspeele, R.; Taerwe, L., 2014, “Influence of Concrete Strength Estimation on the Structural Safety Assessment of Existing Structures”, *Construction and Building Materials*, 62, 77–84, doi:10.1016/j.conbuildmat.2014.03.033.

## 2 STRUCTURAL RELIABILITY

The present chapter introduces concepts and procedures used in the reliability assessment, including time-dependent. Then, several assessment methods to estimate the reliability are reported too.

### 2.1 FAILURE PROBABILITIES AND RELIABILITY INDEX

The failure  $F$  of a system is defined as its inability to achieve or maintain a specified level of performance necessary to fulfil its intended function. In other words, failure occurs when the system cannot perform the required task or meet the expected standards.

System unreliability is referred to the probability of failure  $P_f$  that is the likelihood that a system fails within a certain time period  $T$ . Conversely, the probability of success  $P_s$  is the likelihood that the system will perform its required function without failure during the same time period  $T$ . This success probability can be expressed as

$$P_s = 1 - P_f \quad (2-1)$$

For each type of performance it is possible to define the maximum acceptable probability  $\bar{P}_f$  of failure by considering the consequences of such failure. Failure occurs when this threshold is exceeded.

Every system is described by a set of basic variables, which represent the minimum number of variables required to describe the system's response with respect to the performance under consideration. The set of basic variables is here represented by the vector  $\mathbf{x} = [x_1, \dots, x_n]$ . Basic variables are affected by uncertainty; therefore, they can be represented as random variables  $\mathbf{X}$ , whose statistical characteristics are described by the probability density function Eq. (A-2) or equivalently by the joint probability density function (Appendix A). The set of  $\mathbf{x}$  values describing the unsuccess constitutes the failure domain Eq. (2-2).

$$\Omega_F = \{\mathbf{x}: F\} \quad (2-2)$$

As shown in Eq. (2-3), the failure probability can be calculated based on the probability of occurrence of the  $\mathbf{x}$  values that lie in the failure region  $\Omega_F$ .

$$P_f = \int_{\Omega_F} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \quad (2-3)$$

In reliability assessment, the system capacity can generally be measured using a response scalar function  $g(\mathbf{x})$ . By defining a threshold or limit state  $\bar{g}$  the system's performance is considered satisfactory if  $g(\mathbf{x}) > \bar{g}$ ; otherwise, the performance is deemed unsatisfactory. Therefore, the  $P_f$  can be evaluated by

$$P_f = \int_{R^n} I_F(g(\mathbf{x})) f_X(\mathbf{x}) d\mathbf{x} \quad (2-4)$$

where  $I_F$  is the index function which is 1 if  $g(\mathbf{x}) < \bar{g}$  and 0 otherwise.

For structural members or systems, the  $g(\mathbf{x})$  can be expressed as function of two separate quantities related to the strength of the system  $r(\mathbf{x})$ , and to the effects of the external actions  $s(\mathbf{x})$ . However, resistance and load effects are not deterministic quantities, but random variables with uncertainties arising from structural geometry, material properties, load magnitude and others. Given that  $\mathbf{X}$  is a r.v., previous functions provide r.v. denoted as  $G = g(\mathbf{x})$ ,  $R = r(\mathbf{x})$  and  $S = s(\mathbf{x})$ . By defining failure as the condition where the load effect exceeds the resistance, the probability of failure  $P_f$  can be defined as follows

$$P_f = P(R < S) = \int_0^\infty \int_0^s f_{R,S}(r,s) dr ds \quad (2-5)$$

where  $f_{R,S}(r,s)$  is the joint PDF of  $R$  and  $S$ . Furthermore, if  $R$  and  $S$  are assumed to be statistically independent one has

$$P_f = \int_0^\infty F_R(s) f_S(s) ds = \int_0^\infty [1 - F_S(r)] f_R(r) dr \quad (2-6)$$

where  $F_R$  and  $F_S$  are the CDFs of  $R$  and  $S$ ;  $f_R$  and  $f_S$  are the PDF of  $R$  and  $S$ . The problem can be also illustrated graphically as shown in Figure 2-1.

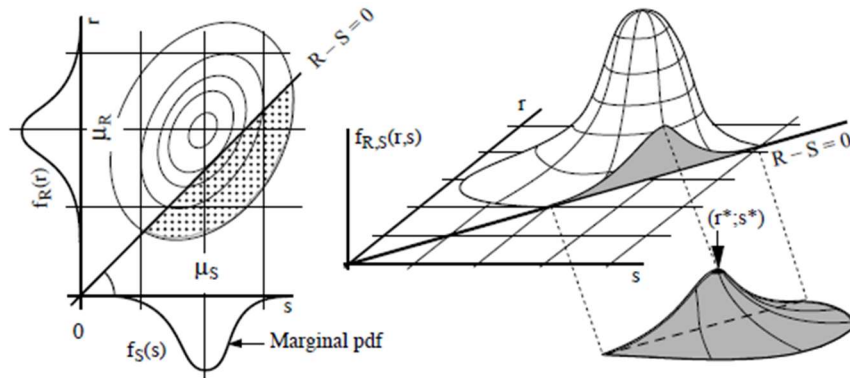


Figure 2-1 Illustration of the joint density function, marginal pdf and failure domain [1]

The most diffused expressions of  $g(\mathbf{x})$  are the safety margin  $z(\mathbf{x})$ , defined by Eq. (2-7), and the safety ratio  $y(\mathbf{x})$ , defined by Eq. (2-8). Given the random nature of  $\mathbf{X}$ , the safety margin and the safety ratio provide r.v. denoted as  $Z = z(\mathbf{x})$  and  $Y = y(\mathbf{x})$ .

$$z(\mathbf{x}) = r(\mathbf{x}) - s(\mathbf{x}) \quad (2-7)$$

$$y(\mathbf{x}) = r(\mathbf{x})/s(\mathbf{x}) \quad (2-8)$$

Eq. (2-5) can be solved analytically under the assumption that  $R$  and  $S$  are independent normally distributed with means values  $\mu_R$  and  $\mu_S$ , and standard deviations  $\sigma_R$  and  $\sigma_S$  respectively. The safety margin  $z(\mathbf{x})$  is also normal variable with mean  $\mu_Z$ , Eq. (2-9), and standard deviation  $\sigma_Z$ , Eq. (2-10).

$$\mu_Z = \mu_R - \mu_S \quad (2-9)$$

$$\sigma_Z = \sqrt{\sigma_R^2 + \sigma_S^2} \quad (2-10)$$

Whit these assumptions, the failure probability  $P_f$  can be simply expressed as

$$P_f = P(Z < 0) = \Phi\left(\frac{0 - \mu_Z}{\sigma_Z}\right) = 1 - \Phi\left(\frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}}\right) \quad (2-11)$$

where  $\Phi$  is the standard normal distribution function. The case  $Z < 0$  corresponds to the failure.

With reference to civil structures, the limit states are classified into two types ([2], [3]): ultimate limit state and serviceability limit state. The first one refers to the partial or whole collapse of a structure; the second one deals with the disruption of structural normal function.

The structural reliability index  $\beta$  is an alternative indicator to express the system safety. It is function of  $P_f$  and it can be defined as in Eq. (2-12). As  $\beta$  increases, the failure probability decreases and the reliability or safety increases.

$$\beta = \Phi^{-1}(1 - P_f) \quad (2-12)$$

The reliability index, also called in this case Cornell reliability index [4], is defined as in Eq. (2-13) and it equals the distance between the mean value of  $Z$  and the origin as shown in Figure 2-2.

$$\beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} = \frac{\mu_Z}{\sigma_Z} \quad (2-13)$$

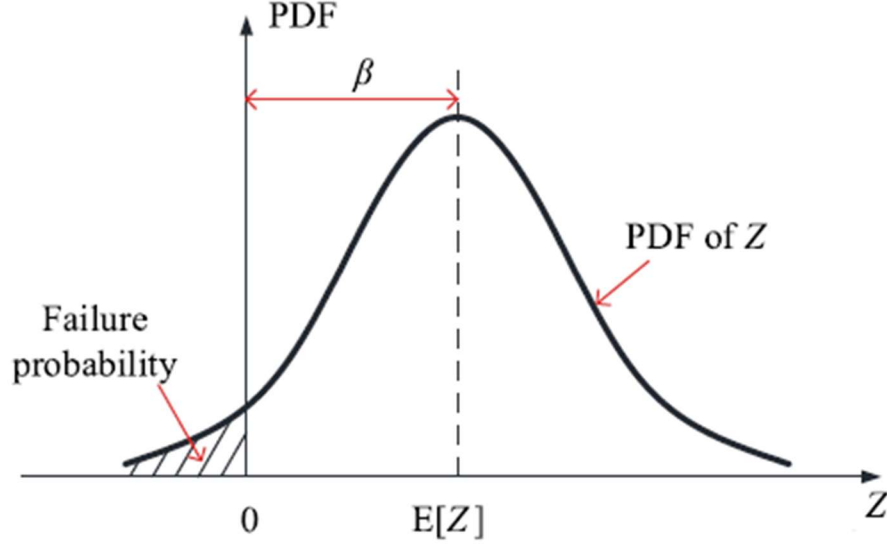


Figure 2-2 Cornell reliability index  $\beta$

Previous calculation method has a disadvantage: the reliability index is not invariant with respect to the formulation of the limit state equation. It follows that the numerical value of  $\beta$  depends on the way in which the problem is mathematically formulated. However, this problem has been overcome by Hasofer and Lind [5] that introduced a generalized reliability index which is invariant with respect to the formulation of the limit state equation. This formulation is commonly applied in reliability analysis. Based on such an approach the uncorrelated normally distributed variables are normalized according to Eq. (2-14). In this way, the limit state equation is transformed in the  $U$ -space and the surface described by this equation divides the  $U$ -space in an unsafe domain  $D_f$  (failure region) and  $D_s$  safe domain (safe region).

$$U_i = \frac{X_i - \mu_i}{\sigma_i} \quad (2-14)$$

Based on (2-14), the basic equation (2-7) can be rewritten as function of the normalized basic variables  $U_R$  and  $U_S$  as

$$z(\mathbf{U}) = \mu_R - \mu_S + U_R \sigma_R - U_S \sigma_S \quad (2-15)$$

According to this formulation  $\beta$  is equal to the shortest distance from the origin to the surface described by the limit state in the space of the normalized variables as shown in Figure 2-3. In Figure 2-3, the coordinates  $[u_R^*, u_S^*]$  correspond to the

design point that is the point of the limit state equation with the highest probability density.

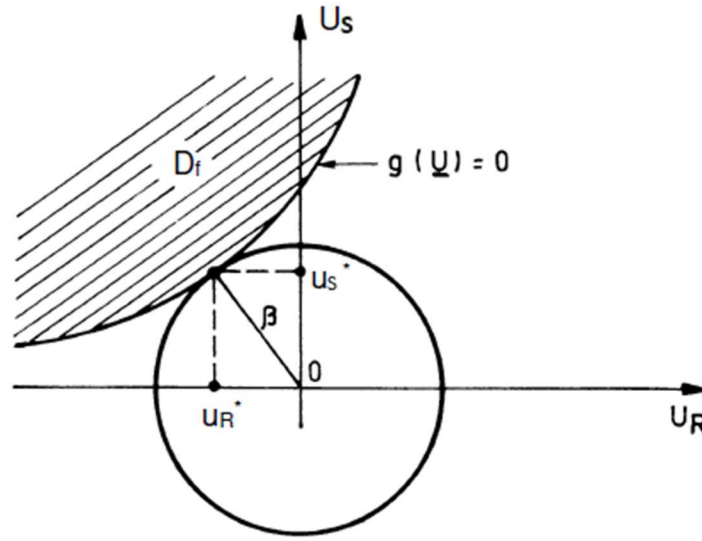


Figure 2-3 Graphical representation of  $\beta$

The advantage of the Hasofer and Lind approach is that it can be applied to more general and complex situation, when compared to the other definitions previously presented.

## 2.2 SAFETY FACTOR AND SEMI-PROBABILISTIC APPROACH

The reliability of a structure can be guaranteed not only by ensuring that the  $P_f$  is under a specified threshold, but also by considering the safety ratio of Eq. (2-8), namely  $y(\mathbf{x})$ . This ratio can be defined in different ways [6], [7]. One possible definition is the central safety ratio  $y_0$  that is the ratio between the mean values of resistance and effect of loads, as shown in Eq. (2-16). However,  $y_0$  does not account for the dispersion of the r.v. Therefore, structures, having the same safety ratio, can have different failure probabilities, as illustrated in Figure 2-4 (a).

$$y_0 = \frac{\mu_R}{\mu_S} \quad (2-16)$$

A more appropriate safety factor is defined in Eq. (2-17) as the ratio between the  $k$ -th fractile of the resistance  $r_k$  and loads effect  $s_k$ . As shown in Figure 2-4 (b), the  $P_f$  of different systems with the same chosen fractile, but varying means and dispersions, is similar. Once decided  $k$ , the safety value  $\bar{y}_k$  is identified and it corresponds to the acceptable probability of failure  $\bar{P}_f$ . The failure happens if  $y_k$  is lower than  $\bar{y}_k$ , as depicted in Figure 2-5.

$$y_k = \frac{r_k}{s_k} \quad (2-17)$$

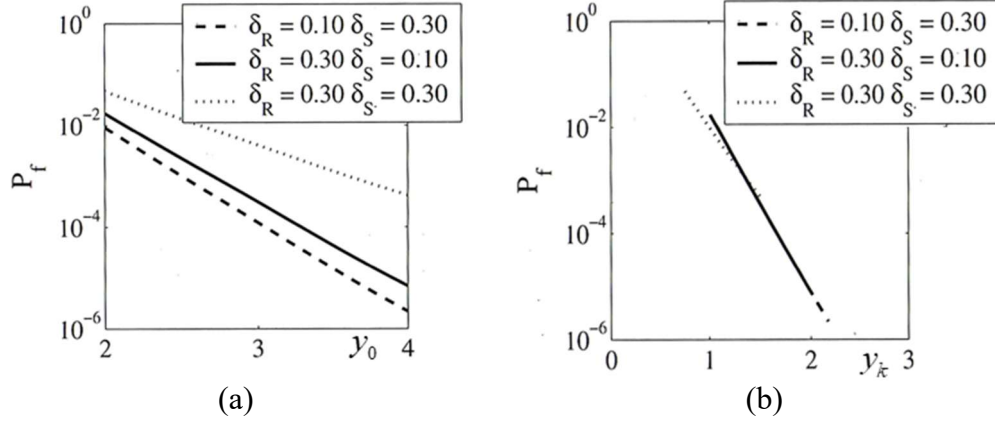


Figure 2-4 Variation of the  $P_f$  function due to different dispersions of resistance and action for (a)  $y_0$  and (b)  $y_k$

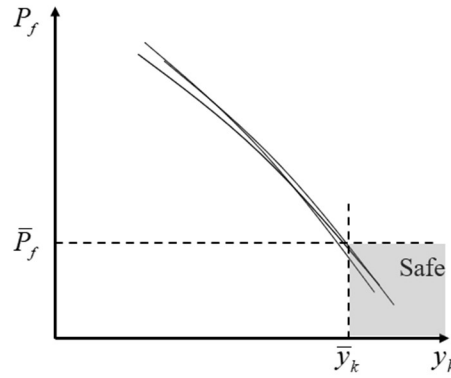


Figure 2-5 Safe and Failure domain in case of fractile safety ratio

The reliability assessment is often carried out by the approximated approach (the semi-probabilistic formulation) proposed by EN-1990 [2]. This method determines deterministic ‘design’ value for strength  $r_d$  and load  $s_d$  by, respectively, dividing and multiplying their characteristic values by calibrated partial factors  $\gamma$ . These parameters are used to compute the safety ratio, as shown in Eq. (2-18). The partial factors depend on the target probability of failure chosen, the descriptors of the probability distribution, the chosen fractile. Partial factor  $\gamma$  are calibrated to ensure that the  $P_f$  remains close to the specified threshold when Eq. (2-18) is equal to 1.

$$y_d = \frac{r_d}{s_d} = \frac{r_k/\gamma_r}{s_d\gamma_s} \quad (2-18)$$

As discussed in Appendix A, a structural system is described by a set of basic random variables  $\mathbf{X}$ . By deriving from  $\mathbf{X}$  the characteristic values and using partial safety factors, the design parameters  $\boldsymbol{\vartheta}_d$  are obtained. In this way the safety ratio can be described as

$$\gamma_d(\boldsymbol{\vartheta}_d) = \frac{r(\boldsymbol{\vartheta}_d)}{s(\boldsymbol{\vartheta}_d)} \quad (2-19)$$

The numerical values of partial factors are contained in EC0 [2], while their background is in its Standard references.

## 2.3 TIME-DEPENDENT RELIABILITY

As discussed in section 2.1, the  $P_f$  of a structure, as defined in Eq. (2-5), represents the probability that the load effect exceeds the resistance. In this equation, both  $R$  and  $S$  are time-invariant; therefore,  $P_f$  is independent of the chosen reference period, as schematically shown in Figure 2-6.

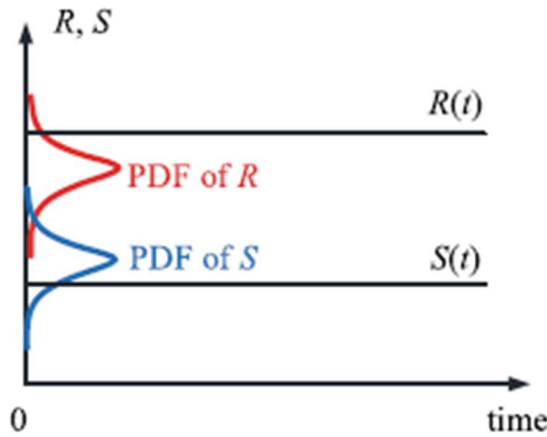


Figure 2-6 Time-invariant example of resistance and load effect [8]

Actually, structures are exposed to environmental effects throughout their service life thus experiencing the deterioration of structural resistance. Additionally, external loads may also vary over the time, further influencing structural response and reliability. As such, a reasonable approach for structural reliability assessment should not disregard the time-variant aspects, especially when considering long reference periods (i.e. 50 years) where the variations of  $R$  and  $S$  over time are non-negligible.

Given a reference period of  $[0, T]$ , the probability of failure  $P_f(0, T)$  is the probability that the load effect  $S$  exceeds the resistance  $R$  at an arbitrary time point within the considered interval. Mathematically, it is expressed as

$$P_f(0, T) = P(S(t) > R(t)), \forall t \in [0, T] \quad (2-20)$$

where  $S(t)$  and  $R(t)$  are the load effect and the resistance at time  $t$ , respectively. The reliability  $L(0, T)$ , called time-dependent reliability, is a complementary part of the  $P_f(0, T)$  and it is expressed as follows

$$L(0, T) = 1 - P_f(0, T) = P(S(t) < R(t)), \forall t \in [0, T] \quad (2-21)$$

Let  $T_f$  be the time of structural failure, the CDF  $T_f$ , denoted as  $F_{T_f}$ , is defined as follows

$$\begin{aligned} F_{T_f} &= P(T_f \leq \tau) = P(\text{failure within } [0, \tau]) = P_f(0, T) \\ &= 1 - L(0, \tau) \end{aligned} \quad (2-22)$$

To solve these equations, both time-variant resistance  $R(t)$  and time-variant load effect  $S(t)$  (Figure 2-7) must be modelled appropriately. The deterioration process of structural resistance  $R(t)$  may be modelled using either stochastic process [8] or deterministic law depending on environmental factors. For concrete structures specific studies address the effect of chloride and carbonation induced corrosion, that are examined in depth throughout chapter 6.

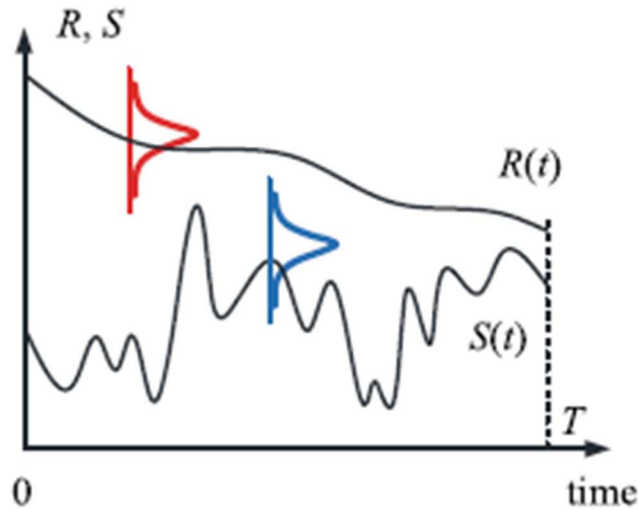


Figure 2-7 Time-variant example of resistance and load effect [8]

Among the time-dependent loads acting on civil structures there are earthquakes, rainfalls, windstorms, traffic actions, and similar phenomena. For instance, traffic load can vary based on vehicle weight and time intervals, both of which are random variables. When observing the load over time, different magnitudes  $S$  can be observed, as shown in Figure 2-8. Over a selected time interval  $[0, T_i]$ , the design

load is the maximum expected value within that interval. As the time interval increases, the probability of exceeding the maximum load also increases.

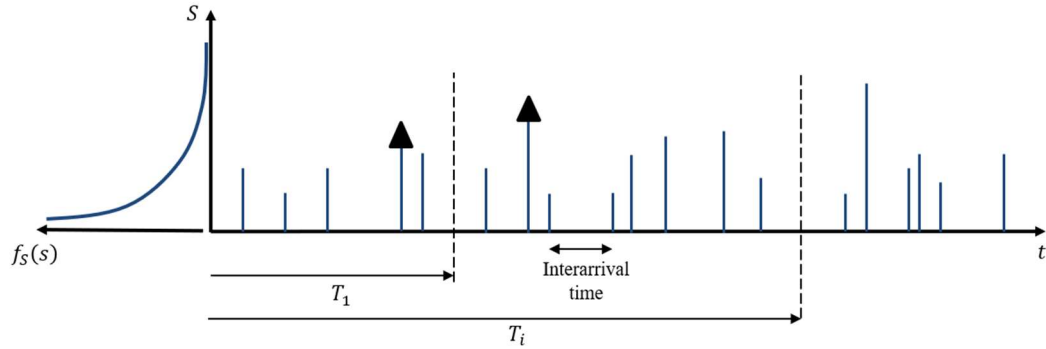


Figure 2-8 Events of a variable load occurred over time

These events can be modelled as stochastic processes [9]-[11]. From a mathematical point of view, a stochastic process is an indexed set of random variables  $\{X(t), t \in T\}$  defined in the sample space  $\Omega$ . At time  $t$  the stochastic variable  $X(t)$  with occurrence  $x(t)$  is described through the CDF distribution function of Eq. (2-23). If  $n$  time instants are considered, for all  $n$ -tuples  $\{t_1, \dots, t_n\}$  and for all  $\{x_1, \dots, x_n\}$ , the CDF joint distribution function is expressed as in Eq. (2-24), while the corresponding joint density function is defined in Eq. (2-25).

$$F_X(x; t) = P(X(t) \leq x), t \in T \quad (2-23)$$

$$F_X(x_1 \dots x_n; t_1 \dots t_n) = P(\{X(t_1) \leq x_1\} \cap \dots \cap \{X(t_n) \leq x_n\}) \quad (2-24)$$

$$f_X(x_1 \dots x_n; t_1 \dots t_n) = \frac{\partial^n F_X(x_1 \dots x_n; t_1 \dots t_n)}{\partial x_1 \dots \partial x_n} \quad (2-25)$$

The time of two subsequent event  $X(t_k) - X(t_{k-1})$ ,  $k = 1, \dots, n$ , is called interarrival time that is characterized by an exponential distribution [11] as reported in Appendix A.

If the time set  $T$  is countable, the process is called discrete stochastic process, instead if  $T$  is continuous the process takes the name of continuous stochastic process. However, the two categories of process can be mutually converted into each other. In the computation of the reliability over time, fixed a reference period  $T$ , the probabilistic behaviour of the maximum load  $S_{max} = \max_{0 \leq t \leq T} S(t)$ , can be described through the Poisson process, Renewal process, and Markov process. A brief overview of the Poisson and Renewal process can be found in Appendix A. The Markov process will be introduced in the chapter of Subset simulation (section 2.5.3.2).

Once the effect of load and resistance over time is known, it becomes possible to compute the probability of failure,  $P_f$ , at each time  $t$ . The time-dependent reliability can be also formulated with the help of several functions that are the hazard rate (or hazard function) and the residual life.

The hazard rate  $h(t)$  is the probability of structural failure at time  $t$  provided that the structure has survived up to time  $t$ . Mathematically, it can be seen as

$$h(t)dt = P(t < failure \leq t + dt | no failure < t) \quad (2-26)$$

From the definition of  $T_f$ , defined in Eq. (2-22),  $h(t)$  can be also expressed as

$$\begin{aligned} h(t)dt &= P(t < T_f \leq t + dt | T_f < t) = \frac{P(t < T_f \leq t + dt)}{P(T_f > t)} \\ &= - \frac{L(0, t + dt) - L(0, t)}{L(0, t)} \end{aligned} \quad (2-27)$$

By integrating both sides from 0 to  $t$  gives

$$L(0, t) = \exp \left[ - \int_0^t h(\tau) d\tau \right] \quad (2-28)$$

By recalling the relationship between the CDF of  $T_f$  and  $L(0, t)$ , Eq. (2-22),  $h(t)$  can be expressed as follows

$$h(t) = \frac{f_{T_f}(t)}{1 - F_{T_f}(t)} \quad (2-29)$$

$h(t)$  can be subdivided into three categories, as shown in Figure 2-9, namely decreasing failure rate (DFR), constant failure rate (CFR), increasing failure rate (IFR). DFR occurs at the early stage of construction, in which the results of the proven test reduce mainly the uncertainties. CFR is often associated with the middle stage of the service life, while IFR is observed at the later stages of the service life where the degradation processes could be significant.

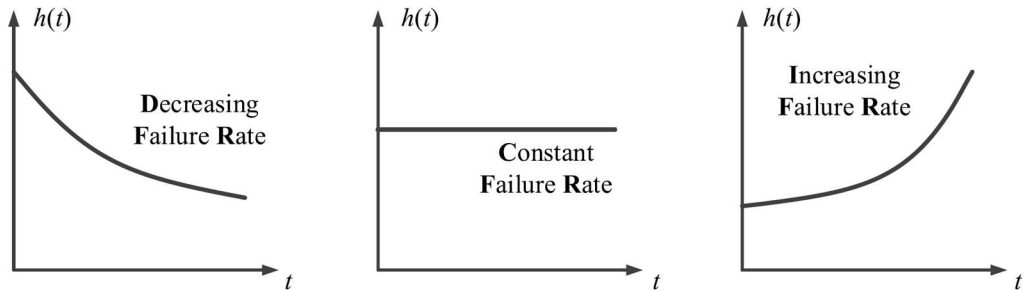


Figure 2-9 Three types of hazard function: DFR, CFR, IFR [8]

$h(t)$  expresses the  $P_f$  in the instant future; more precisely if  $dt$  tends to 0 the limit of  $P_f$  in the time range  $[t, t + dt]$  is equal to  $h(t)dt$ . If  $L(0, t)$  is identified as the reliability in the range  $[0, t]$ , the reliability in the range  $[t, t + dt]$ , assuming that a failure did not happen previously, can be derived as

$$L(t, t + dt) = L(t) \cdot (1 - h(t)dt) \quad (2-30)$$

For infinitesimal time increments the term with  $dt$  can be neglected

If the sequence of load effect is described as a Poisson process with a constant rate of  $\lambda$ , and each load effect is statistically independent and identically distributed,  $h(t)$  is equal to  $\lambda$ . Thus, the reliability  $L(t)$  follows an exponential distribution as described in Eq. (2-31), and the probability of failure is defined as

$$L(t) = e^{-\lambda t}, P_f(t) = 1 - e^{-\lambda t} \quad (2-31)$$

Conversely, if the occurrence rate is time-variant  $\lambda(t)$ , the hazard rate is variable, and the reliability can be defined as

$$L(t) = e^{-\alpha(t)}, \alpha(t) = \int_0^t h(\tau) d\tau \quad (2-32)$$

By considering a structure within a period  $[0, T]$  divided into  $k$  interval of dimension  $dt$  for which  $k$ -th the time step  $t_k = kdt$ , the hazard rate becomes  $h_k = h(t_k)$ . Consequently, the probability of failure is expressed as

$$P_f(t) = 1 - L(t) = 1 - e^{-\alpha(t)} \quad (2-33)$$

From Eq. (2-30), the reliability at  $k$ -th step can be derived from previously step, therefore, the reliability at each step be calculated as follows

$$L_n = \prod_{k=1}^{k=n} (1 - h_k dt) \quad (2-34)$$

## 2.4 RELIABILITY OF STRUCTURAL SYSTEM

Until now the reliability concepts have been applied to an individual structural element. However, real structures or structural systems consist of numerous elements or components that can fail according to several failure modes. Given that an exact analysis of reliability of complex system is almost impossible, it is appropriate to use a simplified model. These models consider system failure in three configurations: series system, parallel system and mixed system.

A system that fails if at least one of its components fails is called a series system or weakest link. Its illustration is given in Figure 2-10. For a system consisted by  $n$

elements, let  $F_i$  the failure event of the  $i$ -th components, its probability of failure is defined by

$$P(F_{sys}) = P\left(\bigcup_{i=1}^n F_i\right) \quad (2-35)$$

where  $F_{sys}$  is the system failure event. If each event is statistically independent the probability of failure becomes

$$P(F_{sys}) = 1 - \prod_{i=1}^n [1 - P(F_i)] \approx \sum_{i=1}^n P(F_i) \quad (2-36)$$

where  $P(F_i)$  are the probabilities of failure of the system's elements. In the case of perfectly correlated elements the  $P(F_{sys})$  is expressed as

$$P(F_{sys}) = \max[P(F_i)] \quad (2-37)$$

Eq. (2-37) and (2-36) are respectively the lower and upper bound of Eq. (2-35).

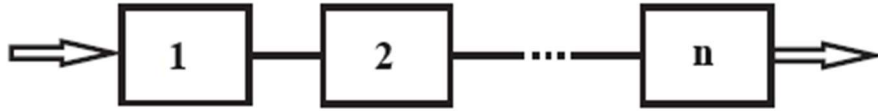


Figure 2-10 Series system

A parallel (or redundant) system, showed in Figure 2-11, is one in which the failure occurs if all the elements fail. In this case, the  $P(F_{sys})$  can be written as

$$P(F_{sys}) = P\left(\bigcap_{i=1}^n F_i\right) \quad (2-38)$$

In the case the failure events are independent,  $P(F_{sys})$  is defined as follows

$$P(F_{sys}) = \prod_{i=1}^n P(F_i) \quad (2-39)$$

By increasing the correlation between the elements, the failure probability of the system increases. In the case of perfect correlation  $P(F_{sys})$  becomes

$$P(F_{sys}) = \min[P(F_i)] \quad (2-40)$$

Eq. (2-40) and (2-39) are respectively the lower and upper bound of Eq. (2-38).

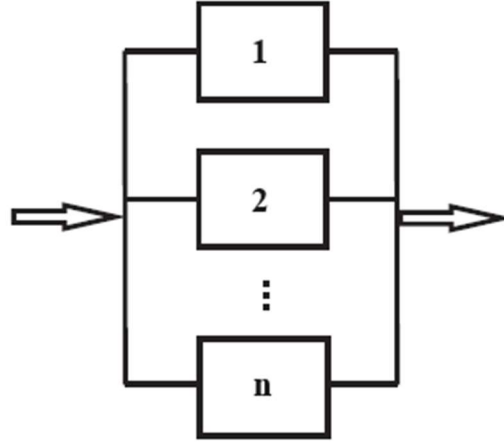


Figure 2-11 Parallel system

The mixed (or combined) system is composed by elements connected in a complex way. By simplifying the system into a combination of series and parallel subsystems, the  $P(F_{sys})$  can be determined by stepwise reduction of the system.

## 2.5 ASSESSMENT METHODS

Several approaches can be employed to estimate the failure probability of a system response. In this section, the probability integral under consideration is reported in Eq. (2-4). The discussion begins with numerical integrations, progresses to the analytical approximation and concludes to the simulation techniques.

### 2.5.1 NUMERICAL INTEGRATION

The numerical integration considers the integral in Eq. (2-4) as a sum of differential contribution. It divides the integration domain  $R^n$ , where  $n$  is the number of variables, into a number of disjoint hypercubes of volume  $\Delta x = \Delta x_1 \dots \Delta x_n$  [12]. It evaluates the integrand function at some representative points and then sums their contribution to approximate the integral. The approximation error, depending on the number of variables  $n$ , is evaluated as  $O(N^{-2/n})$  where  $N$  is number of evaluations of the integrand function. This approach is efficient if the number of variables is small. However, if the integrand function is highly irregular or cannot be solved in closed form, this method cannot be used.

### 2.5.2 ANALYTICAL APPROXIMATION

In this section, an approximated approach called FORM (First-Order Reliability Method) [6], [12], [13] is used to solve Eq. (2-4). The method is suitable in the case of an  $n$ -dimensional problem. By assuming that random variables  $\mathbf{X}$  are Gaussian distributed, the failure region is approximated by a linear space as showed in Figure 2-12. As it is shown in section. 2.1, the reliability, expressed by the  $\beta$  index, is equal to the shortest distance from the origin to the surface described by the limit state function  $g(\mathbf{x})$ .

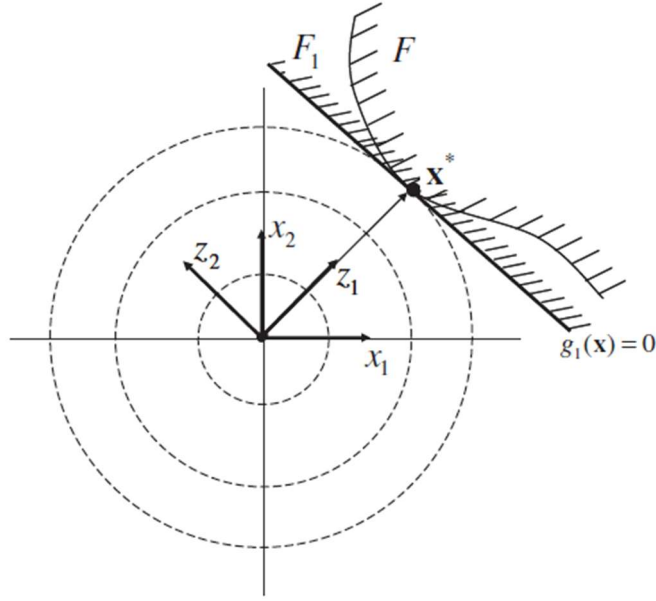


Figure 2-12 FORM [12]

By considering a limit state function  $g(\mathbf{x}) = a_0 + \sum_{i=1}^n X_i a_i$ , where  $X_i$  are random variables ( $i = 1 \dots n$ ) and  $a_i$  are constants and by transforming the limit state equation into the normalized space, the reliability index can be expressed as

$$\beta = \frac{a_0 + \sum_{i=1}^n \mu_i a_i}{\sqrt{\sum_{i=1}^n (a_i \sigma_i)^2}} \quad (2-41)$$

The coordinates of the point knowns as the checking point  $\mathbf{x}^*$ , which lies on the line  $g(\mathbf{x}) = 0$  and that yields to the shortest distance from the origin to  $g(\mathbf{x}) = 0$  in the original space, is determined by

$$x_i^* = \mu_i - \frac{\beta a_i \sigma_i^2}{\sqrt{\sum_{i=1}^n (a_i \sigma_i)^2}} \quad (2-42)$$

The normal vector to  $g(\mathbf{x}) = 0$  at the point  $\mathbf{x}^*$  is denoted as  $\boldsymbol{\alpha}$  and its components are defined as

$$\alpha_i = \frac{a_i \sigma_i}{\sqrt{\sum_{i=1}^n (a_i \sigma_i)^2}} \quad (2-43)$$

The probability of failure is obtained as follows

$$P_f = \Phi(-\beta) \quad (2-44)$$

If the limit state  $g(\mathbf{x})$  is nonlinear, the  $\beta$  is obtained linearizing  $g(\mathbf{x})$  and therefore it is equal to the shortest distance from the origin to the linearized surface  $\tilde{g}(\mathbf{x}) = 0$  as showed in Figure 2-13. This linearization is performed by the first-order Taylor expansion of  $g(\mathbf{x})$  in the point  $\mathbf{x}'$  for which the limit state becomes

$$g(\mathbf{x}) = \tilde{g}(\mathbf{x}) = g(\mathbf{x}') + \sum_{i=1}^n (X_i - x_i') \frac{\partial g(\mathbf{x}')}{\partial X_i} \quad (2-45)$$

In the U-Space Eq. (2-45) becomes

$$g(\mathbf{u}) = \tilde{g}(\mathbf{u}) = g(\mathbf{u}') + \sum_{i=1}^n (U_i - u_i') \frac{\partial g(\mathbf{u}')}{\partial U_i} \quad (2-46)$$

$\beta$  can be written as follows

$$\beta = \frac{-\sum_{i=1}^n \frac{\partial g(\mathbf{u}')}{\partial U_i} u_i'}{\sqrt{\sum_{i=1}^n \left[ \frac{\partial g(\mathbf{u}')}{\partial U_i} \right]^2}} = \frac{-\sum_{i=1}^n \frac{\partial g(\mathbf{x}')}{\partial X_i} (x_i' - \mu_i)}{\sqrt{\sum_{i=1}^n \left[ \frac{\partial g(\mathbf{x}')}{\partial X_i} \sigma_i \right]^2}} \quad (2-47)$$

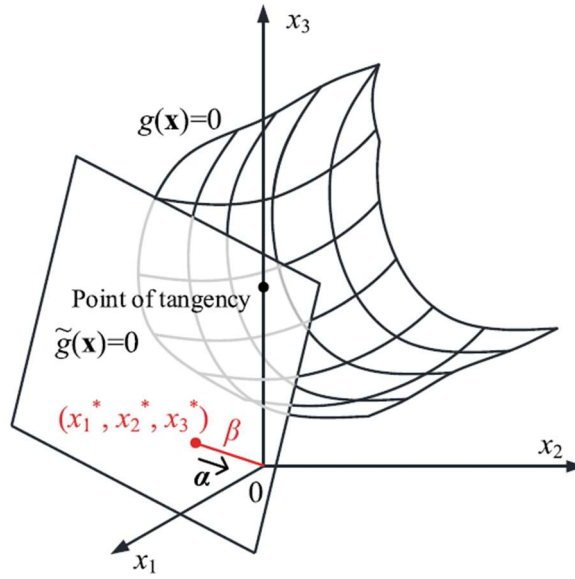


Figure 2-13 Non-linear limit state equation [8]

Eq. (2-47) depends on the point  $\mathbf{x}'$  where the Taylor expansion is performed. Therefore, different choices of  $\mathbf{x}'$  lead to different reliability indices. To solve this problem and to improve the accuracy of the estimating  $\beta$ , two different methods can be used and are only mentioned here. The first is based on the use of a Lagrange multiplier; this approach results to be difficult in many cases. The second is based on an iteration-based approach.

When variables are not normally distributed, the analysis can be performed by transforming the non-normal variables  $X_i$  into a corresponding normal variable  $X'_i$ . This transformation is accomplished by determining the mean value and standard deviation of  $X'_i$  such that the probability distributions of  $X'_i$  and  $X_i$  are identical at the checking point. The transformation method is called the normal tail transform. Two methods also employed for this purpose and are the Roseblatt transformation method and the Nataf transformation method [8]. When, the variables are independent, both methods give

$$y_i = \Phi^{-1}[F_i(X_i)], i = 1, \dots \quad (2-48)$$

where  $y_i$  is a standard normal variable. Therefore, the  $\beta$  can be calculated as described previously.

The FORM is useful for simple problem; however determining the shortest point can be challenging due to the potential presence of local minimum in the domain. Moreover, it is not accurate when the limit state function is far from linear. A more

efficient analytical method is the SORM (second order reliability method) [6] not treated in this dissertation. Both methods rely on identifying a design point to estimates the failure probability. However, their efficiency decreases with an increasing number of variables involved and with the degree of nonlinearity of the system response [14], [15].

### 2.5.3 SIMULATION TECHNIQUES

Two simulation methods are discussed in this section: the Monte Carlo (MC) Simulation and the Subset Simulation.

#### 2.5.3.1 MONTE CARLO SIMULATION

The Monte Carlo simulation [12], [16], [17] is valuable for several reasons. It imposes no restrictions on the shape of  $g(\mathbf{x})$ , on the possible distribution of  $\mathbf{X}$ , or on the topological characteristics of the failure domain, which can include disjoint subdomains and/or complex boundaries. Recall Eq. (2-4) once again

$$P_f = \int_{R^n} I_F(g(\mathbf{x})) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} = E[r(\mathbf{x})], \quad r \in \{0, 1\} \quad (2-49)$$

In the Monte Carlo (MC) simulation, the probability of failure is estimated by sampling from the pdf  $f_{\mathbf{X}}$ , of the basic random variables  $\mathbf{X}$ ,  $N$  realizations  $\mathbf{X}_k$ .  $\mathbf{X}_k$  are independent and identically distributed (i.i.d). The integral can be estimated by the sample average as

$$\bar{P}_f = \frac{1}{N} \sum_{k=1}^N R_k \quad (2-50)$$

where  $\bar{P}_f$  is the estimator of the failure probability, while  $R_k$  is given below.

$$R_k = r(\mathbf{X}_k) \quad (2-51)$$

If  $r(\mathbf{x})$  is defined as the index function  $I_F(g(\mathbf{x}))$ , the estimator of the failure probability  $\bar{P}_f$  can be simply seen as the ratio of the number of failures samples to the total number of samples  $N$ , as given in Eq. (2-52).

$$\bar{P}_f = E[r(\mathbf{x})] = \frac{1}{N} \sum_{k=1}^N r_k = \frac{1}{N} \sum_{k=1}^N I_F(g(\mathbf{x}_k)) = \frac{n_{failure}}{N} \quad (2-52)$$

Two key properties of the estimator are that it is ‘unbiased’ and ‘convergent’. Unbiased means that as  $N \rightarrow \infty$ , the expected value of  $\bar{P}_f$  equals  $P_f$ . This implies

that even when performing multiple simulations runs, the values of  $\bar{P}_f$  will scatter around the (unknown) target value  $P_f$ . The dispersion of these values decreases as  $N$  increases. Convergent means that the variance of the estimator, given by Eq. (2-53), tends to zero as  $N \rightarrow \infty$ .

$$\text{var}[\bar{P}_f] = \frac{1}{N^2} \sum_{k=1}^N \text{var}[R_k] = \frac{\text{var}[r(\mathbf{X})]}{N} \quad (2-53)$$

To assess the accuracy of the estimator, it is more convenient to refer to its  $c_v$ , given by

$$c_v[\bar{P}_f] = \frac{\sqrt{\text{var}[r(\mathbf{X})]}}{E[r(\mathbf{X})]} \frac{1}{\sqrt{N}} \quad (2-54)$$

Figure 2-14 illustrates these properties. In Figure 2-14 (a), the example estimator  $\tilde{J}_N$  converges toward the target value and exhibits reduced fluctuation as number of samples  $N$  increases. Figure 2-14 (b) shows that for 100 independent runs, the estimator varies between the runs but remains centred around the target value. Moreover, as  $N$  increases, the scattering of runs decreases.

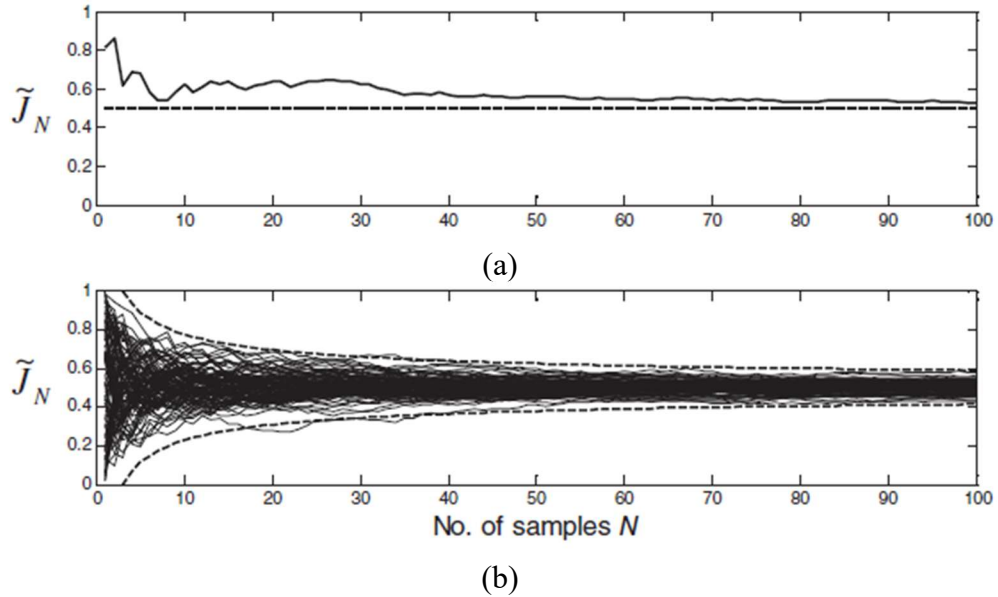


Figure 2-14 Example of the estimator properties (a) single run; (b) 100 runs [12]

The number of necessary realizations,  $N$ , is function either of the number of significant digits with which one wants to calculate  $P_f$  or of the relative accuracy. By considering a binomial distribution, Eq. (2-53) and Eq. (2-54) can be rewritten as follow

$$var[\bar{P}_f] = \frac{1}{N} P_f (1 - P_f) \quad (2-55)$$

$$c_v[\bar{P}_f] = \sqrt{\frac{1 - P_f}{P_f N}} \quad (2-56)$$

As  $P_f \ll 1$  this leads to

$$c_v[\bar{P}_f] = \frac{1}{\sqrt{P_f N}} \quad (2-57)$$

By defining a threshold of the failure probability the corresponding quantile  $g_\alpha$ , for which  $P(G > g_\alpha) = \alpha$ , it can be expressed as

$$g_\alpha = \bar{F}_G^{-1}(\alpha) \quad (2-58)$$

where  $\bar{F}_G^{-1}()$  is the inverse of the complementary cumulative distribution function  $\bar{F}_G$  of  $G$ . Commonly, the probability of failure sought is very low, therefore, it is difficult to have a sufficient number of samples for a statistic estimation of the corresponding quantile. The c.v. of estimate quantile  $\bar{g}_\alpha$ , expressed in (2-59), is showed in Figure 2-15. The horizontal bars are the quantile at the failure probability (dot). For failure minor than 0.01 the probability and quantile estimates are, similar, while below it they significantly differ.

$$c_v[\bar{g}_\alpha] = \sqrt{\frac{1 - \alpha}{\alpha N}} \quad (2-59)$$

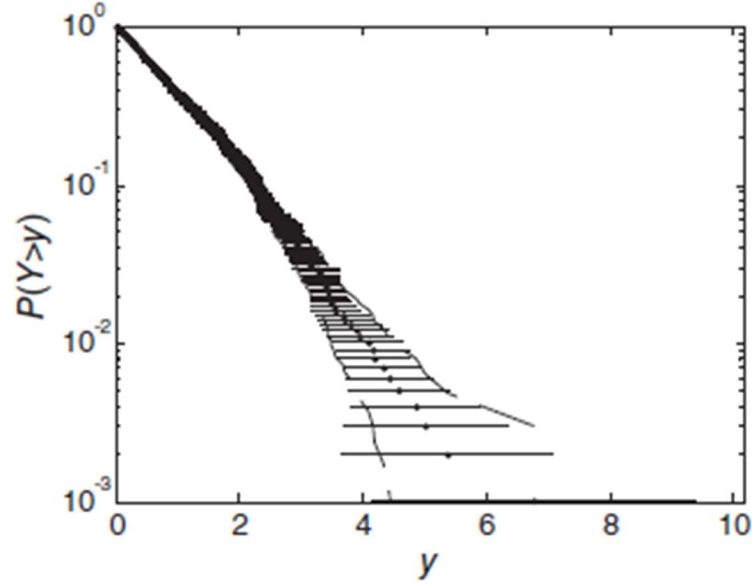


Figure 2-15 Failure probability and quantile estimation [12]

MC simulation is not efficient for estimating small failure probabilities or rare events. Intuitively, when failure is small, most of the samples do not fail. Increasing the number of samples may solve the problem but on the other hand it increases the computational time and cost of the analysis.

### 2.5.3.2 SUBSET SIMULATION

Subset simulation is an advanced and efficiently simulation technique that has been proposed by [12]. It is robust towards the number of uncertain parameters, and efficient in computing small probabilities. A failure event  $F$ , defined as an intersection of decreasing sequence of  $M$  intermediate failure events, is expressed as in Eq. (2-60). For example, if the failure of a structure is defined as the exceedance of an action  $S$  over the resistance  $R$ , then the sequence of failure events can be defined as  $F_i = \{R_i < S\}$ , where  $F_1 < F_2 < \dots < F_M = F$ .

$$F = \bigcap_{i=1}^M F_i \quad (2-60)$$

By using the conditional probability (see Appendix A), the  $P_f$ , defined in Eq. (2-61), is a product of a sequence of failure event.

$$P_f = P\left(\bigcap_{i=1}^M F_i\right) = \prod_{i=1}^M P(F_i|F_{i-1}) \quad (2-61)$$

Where for  $i = 1$   $F_0$  is the certain event. Basically, the more the searched  $P_f$  is small, the more the failure event is rare, thus more samples are required. By selecting

intermediate failure events adequately, the conditional probabilities  $P(F_i|F_{i-1})$  can be sufficiently large and can be efficiently evaluated by simulation procedures. The intermediate ‘level intermediate’  $p_0$  is commonly of the order of 0.1.

Let  $\mathbf{X}$  be the vector collecting the basic random variables that are independent with a joint PDF equal to  $q(\mathbf{x})$ . The scalar response of the system is determined as  $Y = h(\mathbf{X})$ .

The Subset simulation estimates a sequence of thresholds  $b$  corresponding to a set of predefined exceedance probabilities  $P(Y > b)$ . The procedure starts from simulation level 0, in which MC Simulation is performed, followed by subsequent simulation level  $i = 1 \dots M - 1$  in which Monte Carlo Markov Chain (MCMC) sampling technique is used [18].

As previously mentioned, each level  $i$  of the Subset simulation exploits the MCMC to sample from the conditional PDF  $q(\mathbf{x}|F_i)$ . Before delving into the principles of MCMC, it is important to introduce the fundamental theory of Markov chains.

A stationary discrete-time process  $\{X_t, t \in N\}$  with conditional PDF  $q(\mathbf{x}|F_i)$  is called a Markov chain or Markov process if the state  $X_{n+1}$  depends on the previous state  $X_n$  only. This condition is described by Eq. (2-62).

$$P\left(X_{n+1} \in A \mid \bigcap_{t \leq n} X_t = x_t\right) = P(X_{n+1} \in A | X_n = x_n) \quad (2-62)$$

where  $A$  is any event in the outcome space  $X_t$ . The process that defines the transition between two subsequent states  $X_n$  and  $X_{n+1}$  is called transition PDF  $p$  that is independent of  $n$ . Starting from an initial state that may or may not follow the target distribution, the Markov chain will asymptotically converge to the target (stationary) distribution, provided that the chain is aperiodic and irreducible [16], [19]. Ensuring these conditions is relatively straightforward for most MCMC algorithms: one must select a transition probability distribution that assigns a non-zero probability to ensure aperiodicity and irreducibility. The initial period during which the Markov chain approaches its stationary distribution is known as the burn-in period. Furthermore, the samples generated by the Markov chain will exhibit correlation based on the specific choice of the transition PDF, reflecting the inherent correlation of the Markov process. For more detailed information, it is possible to refer to [20].

As showed previously, in Subset simulation at the stage  $i$  the MCMC sampling generates a sample sequence of  $N_C$  Markov Chain each starting from the seeds  $\mathbf{X}_{j0}^{(i)}$  of previous step. Since all seeds  $\mathbf{X}_{j0}^{(i)}$  are distributed according to  $q(\cdot | F_i)$ , the chains have already reached their stationary states at the beginning and no burn-in period is necessary. All the subsequent states of the Markov chains are distributed according to  $q(\cdot | F_i)$ . The Metropolis-Hasting (M-H) algorithm [21] is the most

widely used MCMC method in the literature. The M-H method is here described analysing the transition from a state  $\mathbf{X}_{jk}^{(i)}$  to the next state  $\mathbf{X}_{j,k+1}^{(i)}$  as follows:

1. Generate  $U$  as uniformly distributed  $[0,1]$  and candidate sample for  $\mathbf{X}^*$  from proposal PDF  $p^*(\cdot | \mathbf{X}_{jk})$
2. Calculate the acceptance probability  $r$  defined as follows

$$r = \min \left\{ 1, \frac{p^*(\mathbf{X}_{jk}^{(i)} | \mathbf{X}^*) q(\mathbf{X}^*)}{p^*(\mathbf{X}^* | \mathbf{X}_{j0}^{(i)}) q(\mathbf{X}_{jk}^{(i)})} \right\} \quad (2-63)$$

3. Accept or reject  $\mathbf{X}^*$

$$\mathbf{X}_{j,k+1}^{(i)} = \begin{cases} \mathbf{X}^* & \text{if } U < r \\ \mathbf{X}_{jk}^{(i)} & \text{otherwise} \end{cases} \quad (2-64)$$

Introducing the Markov process, the steps of Subset Simulation are now detailed. Here,  $N$  represents the number of samples at each level,  $N_c = Np_0$  denotes the number of chains and  $N_s = p_0^{-1} = \frac{N}{N_c}$  defines the number of samples per chain. Simulation at Level 0 is divided into the following key points:

- Generate  $N$  i.i.d. samples  $\{\mathbf{X}_k^{(0)} : k = 1, \dots, N\}$ , according to the parameter PDF  $q$  and then calculate the values of response  $\{Y_k^{(0)} = h(\mathbf{X}_k^{(0)}) : k = 1, \dots, N\}$ ;
- Sort in ascending  $\{Y_k^{(0)} : k = 1, \dots, N\}$  order to give the list of  $\{b_k^{(0)} : k = 1, \dots, N\}$  values, that gives the estimate of  $b$  corresponding to the exceedance probability  $p_k^{(0)} = P(Y > b)$  defined in (2-65). Plotting  $\{(b_k^{(0)}, p_k^{(0)}) : k = 1, \dots, N - N_c\}$  gives the CDF of  $Y$  with probability ranging from  $(1 - N^{-1})$  to  $p_0$

$$p_k^{(0)} = \frac{N - k}{N}, k = 1, \dots, N \quad (2-65)$$

- Set  $b_1 = b_{N-N_c}^{(0)}$

$$b_1 = b_{N-N_c}^{(0)} \quad (2-66)$$

Let  $\{\mathbf{X}_{j0}^{(1)} : j = 1, \dots, N_c\}$  be the samples of  $\mathbf{X}$  corresponding to  $\{b_{N-N_c+j}^{(0)} : j = 1, \dots, N_c\}$ . These samples are used as ‘seeds’ for generating additional samples conditional on  $F_1 = \{Y > b_1\}$  at simulation level 1.

The estimate probability at Level 0 is given by (2-67), while its c.v. is calculated according to Eq. (2-56).

$$P(F_0) = \tilde{P}_0 = \frac{1}{N} \sum_{k=1}^N I_{F_0}(\mathbf{X}_k^{(0)}) \quad (2-67)$$

The simulation Level  $i = 1, \dots, M - 1$  (MCMC) is subdivided into the following key points:

- From each seed  $\mathbf{X}_{j0}^{(i)}$  ( $j = 1, \dots, N_c$ ) belonging to the last level  $i - 1$ , MCMC is used to generate  $N_s$  conditional samples  $\{\mathbf{X}_{jk}^{(i)} : k = 1, \dots, N_s\}$  with conditional PDF  $q(\cdot | F_i)$ . This gives  $N_c$  Markov chains, each with  $N_s$  samples. For each  $i$ -th level the total number of samples is still equal to  $N$ . The seeds are discarded after use.
- Let  $Y_{jk}^{(i)} = h(\mathbf{X}_{jk}^{(i)})$  be the response function calculated during the MCMC. Sort  $\{Y_{jk}^{(i)} : j = 1, \dots, N_c; k = 1, \dots, N_s\}$  in ascending order to give the list  $\{b_k^{(i)} : k = 1, \dots, N\}$ . The value  $b_k^{(i)}$  gives the estimate of  $b$  corresponding to exceedance probability  $p_k^{(i)} = P(Y > b)$  defined in Eq. (2-68). Plotting  $\{(b_k^{(i)}, p_k^{(i)}) : k = 1, \dots, N - N_c\}$  gives the CCDF estimate of  $Y$  with probability ranging from  $p_0^{(i)}(1 - N^{-1})$  to  $p_0^{(i+1)}$ .

$$p_k^{(i)} = p_0^{(i)} \frac{N - k}{N}, k = 1, \dots, N \quad (2-68)$$

- Set

$$b_{i+1} = b_{N-N_c}^{(i)} \quad (2-69)$$

Let  $\{\mathbf{X}_{j0}^{(i+1)} : j = 1, \dots, N_c\}$  be the  $N_c$  samples of  $\mathbf{X}$  corresponding to  $\{b_{N-N_c+j}^{(i)} : j = 1, \dots, N_c\}$ . These samples are used as ‘seeds’ for generating additional samples conditional on  $F_{i+1} = \{Y > b_{i+1}\}$  at simulation level 1. This step is omitted if  $i = M - 1$ , because it is the last level.

At each  $i$ th conditional level,  $1 \leq i \leq M - 1$ , let  $\{\mathbf{X}_k^{(i)}, k = 1, \dots, N\}$  be the Markov samples with distribution  $q(\cdot | F_i)$ , the conditional probabilities  $P(F_{i+1} | F_i)$  are shown in the following equation:

$$P(F_{i+1}|F_i) \approx \tilde{P}_{i+1} = \frac{1}{N} \sum_{k=1}^N I_{F_{i+1}}(\mathbf{X}_k^{(i)}) \quad (2-70)$$

By combining Eq. (2-61), (2-67), (2-70), the estimator of the failure probabilities is obtained as shown in Eq. (2-71)

$$P_f \approx \tilde{P}_f = \prod_{i=1}^M P_i \quad (2-71)$$

To better understand the Subset Simulation an illustrate examples is proposed in Figure 2-16

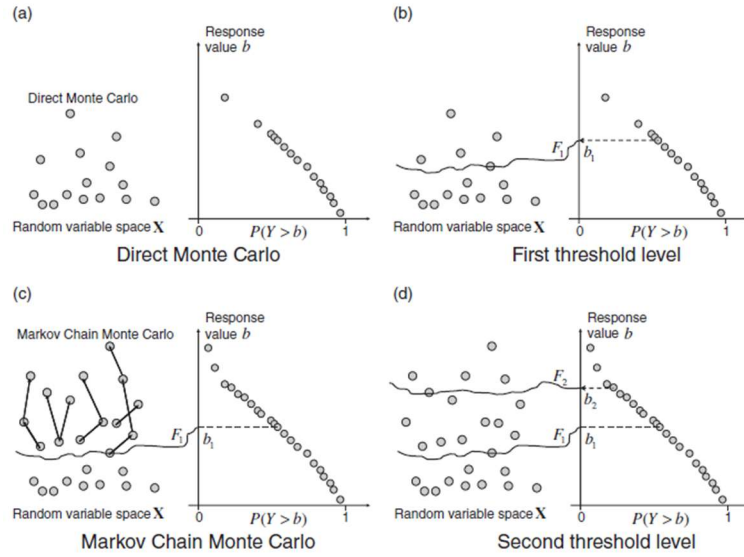


Figure 2-16 Subset Simulation illustration: (a) Level 0 performed with Direct MC Simulation; (b) First threshold level, (c) MCMC chains, (d) Second threshold level [12]

For each  $i$ -th conditional level, the coefficient of variation  $\delta_i$  of  $\tilde{P}_i$ , given in [22], is

$$\delta_i = \sqrt{\frac{1 - P_i}{P_i N} (1 + \gamma_i)} \quad (2-72)$$

where  $\gamma_i$  is the correlation factor given in Eq. (2-63), in which  $\rho_i(k)$ ,  $k = 1, \dots, N/N_c - 1$ , is the average  $k$ -lag auto-correlation coefficient of the chains that can be estimated from the samples [22]. The ratio factor shown in Eq. (2-72) is the familiar one for the square of the c.v. of MC simulation, already proposed in Eq. (2-56), with  $N$  i.i.d. samples. The efficiency of a Markov process using a dependent sample ( $\gamma_i > 0$ ) is therefore lower compared to the independent samples case ( $\gamma_i = 0$ ). Smaller values  $\gamma_i$  entail a higher efficiency.

$$\gamma_i = 2 \sum_{k=1}^{\frac{N}{N_c}-1} \left(1 - \frac{kN_c}{N}\right) \rho_i(k) \quad (2-73)$$

The c.v. of Eq. (2-71) denoted here by  $a_i$  may be approximately bounded as

$$a_L < a_i < a_U \quad (2-74)$$

where  $a_U$ , shown in Eq. (2-75), is the upper bound corresponding to the case when the conditional probability estimators are fully correlated, while  $a_L$ , defined in Eq. (2-76), is the lower bound in the uncorrelated case.

$$a_U = \left( \sum_{k=1}^i \sum_{j=1}^i \delta_k \delta_j \right)^{1/2} \quad (2-75)$$

$$a_L = \left( \sum_{j=1}^i \delta_j^2 \right)^{1/2} \quad (2-76)$$

To estimate the number of samples required to achieve a given accuracy in the probability of failure, consider  $P(F_1) = P(F_i|F_{i-1}) = p_0$ , and assume the  $\delta_i$  equal across all levels. By using Eq.(2-72) and (2-75), and by noting that the number of simulations levels to achieve the target failure probability is  $m = \log P_f / \log p_0$ , the total number of samples  $N_T$  to achieve a given c.v. is roughly

$$N_T \approx mN = |\log P_f|^r \frac{(1 + \gamma)(1 - p_0)}{p_0 |\log p_0|^r \delta^2} \quad (2-77)$$

where  $r \leq 3$  depends on the correlation of the conditional probability estimators. Thus, for a fixed  $p_0$  and  $\delta$ ,  $N_T$  is proportional to  $|\log P_f|^r$ . Compared with MC simulation, where  $N_T$  is proportional to  $1/P_f$ , this implies a substantial improvement in the efficiency when estimating small probabilities.

## 2.6 REFERENCES

- [1] Schneider, J., Vrouwenvelder, T., 1997. Introduction to safety and reliability of structures, 3rd ed. International Association for Bridge and Structural Engineering (IABSE), Zurich, Switzerland. <https://doi.org/10.2749/sed005>
- [2] CEN, 2002. EN 1990 Eurocode 0 – Basis of Structural Design, Brussel
- [3] Ellingwood, B., Development of a probability based load criterion for American National Standard A58.
- [4] Cornell, C.-A., 1969. A Probabilistic-Based Structural Code, ACI Journal
- [5] Hasofer, A., Lind, N., 1974. An exact and invariant first order reliability format, Proc. ASCE, J. Eng. Mech. Div.
- [6] Ditlevsen, O., Madsen, H.O., Ditlevsen, O., 1999. Structural reliability methods, Repr. ed. Wiley, Chichester.
- [7] Soronsen, J.-D., 2004. Notes in Structural Reliability Theory and Risk Analysis
- [8] Wang, C., 2021. Structural Reliability and Time-Dependent Reliability, Springer Series in Reliability Engineering. Springer International Publishing, Cham. <https://doi.org/10.1007/978-3-030-62505-4>
- [9] Nakagawa, T., 2011. Stochastic Processes: with Applications to Reliability Theory, Springer Series in Reliability Engineering. Springer London, London. <https://doi.org/10.1007/978-0-85729-274-2>
- [10] Lai, C.-D., Xie, M., 2006. Stochastic ageing and dependence for reliability. Springer, New York NY [Heidelberg].
- [11] Beichelt, F., 2006. Probability Theory. STOCHASTIC PROCESSES.
- [12] Au, S., Wang, Y., 2014. Engineering Risk Assessment with Subset Simulation, 1st ed. Wiley. <https://doi.org/10.1002/9781118398050>
- [13] Papaioannou, I., Straub, D., 2021. Variance-based reliability sensitivity analysis and the FORM  $\alpha$  -factors. Reliability Engineering & System Safety 210, 107496. <https://doi.org/10.1016/j.ress.2021.107496>
- [14] Schuëller, G.I., Pradlwarter, H.J., Koutsourelakis, P.S., 2004. A critical appraisal of reliability estimation procedures for high dimensions. Probabilistic Engineering Mechanics 19, 463–474. <https://doi.org/10.1016/j.probengmech.2004.05.004>
- [15] Katafygiotis, L.S., Zuev, K.M., 2008. Geometric insight into the challenges of solving high-dimensional reliability problems. Probabilistic Engineering Mechanics 23, 208–218. <https://doi.org/10.1016/j.probengmech.2007.12.026>
- [16] Rubinstein, R.Y., Kroese, D.P., 2017. Simulation and the Monte Carlo method, Third edition. ed, Wiley series in probability and statistics. John Wiley & Sons, Inc, Hoboken, New Jersey.
- [17] Haldrar, A., Mahadevan, S., 2000. Probability, reliability, and statistical methods in engineering design. John Wiley, New York; Chichester [England].

- [18] Papaioannou, I., Betz, W., Zwirgmaier, K., Straub, D., 2015. MCMC algorithms for Subset Simulation. *Probabilistic Engineering Mechanics* 41, 89–103. <https://doi.org/10.1016/j.probengmech.2015.06.006>
- [19] Tierney, L., 1994. Markov Chains for Exploring Posterior Distributions. *Ann. Statist.* 22. <https://doi.org/10.1214/aos/1176325750>
- [20] Besag, J., Green, P., Higdon, D., Mengersen, K., 1995. Bayesian Computation and Stochastic Systems. *Statist. Sci.* 10. <https://doi.org/10.1214/ss/1177010123>
- [21] Hastings, W.K., 1970. Monte Carlo sampling methods using Markov chains and their applications.
- [22] Au, S.-K., Beck, J.L., 2001. Estimation of small failure probabilities in high dimensions by subset simulation. *Probabilistic Engineering Mechanics* 16, 263–277. [https://doi.org/10.1016/S0266-8920\(01\)00019-4](https://doi.org/10.1016/S0266-8920(01)00019-4)

## 3 POST-TENSIONED BRIDGES

### 3.1 INTRODUCTION

Aiming at enhancing the performance of structures, prestressing can be generally defined as the application of a preload before service loads. This technique is primarily used in the design of concrete structures. The goal of prestressing concrete (PC) is mainly to reduce or even eliminate tensile stresses in the material by introducing a compressive force.

At its earliest application to beams, the primary objective of prestressed concrete was to completely eliminate tensile stresses by applying sufficient axial force. While, as the knowledge of this construction technique advanced over time, the elimination of tensile stresses requirement has been considered overly restrictive. In modern design practices, some degree of tensile stress, and even limited cracking, is currently allowed. The reduction or elimination of tension and cracking in prestressed concrete enables the full cross-sectional properties of the material to be used to resist bending and deflection. Maintaining an uncracked section allows for the design of longer, thinner, and more cost-effective flexural components.

Additionally, prestressing enhances the structural serviceability by allowing engineers to control the number and width of cracks, as well as the deflection of the element, through adjustments in the level of compressive prestress. In fact, partial prestressing makes it possible to design elements with zero deflection under specific combinations of prestress and external loads. This approach is preferable to full prestressing, which can cause undesirable upward camber (excessive upward curvature) in the structure.

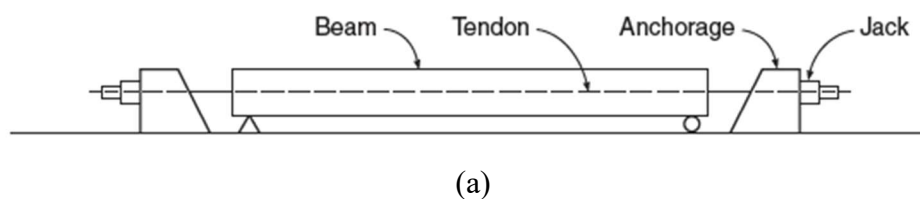
The creator of practical prestressed concrete was the French engineer Eugene Freyssinet [1], who started to use high-strength steel wire for prestressing concrete in 1928. Previous attempts to produce prestressed concrete using normal reinforcement had been unsuccessful. In 1940 he introduced a system for prestressing wedge-anchored high strength steel cables. An example of typical Freyssinet's design is the bridge over the Marne river in Luzancy Figure 3-1, built in 1941 and composed by I-shaped segment pre-cast that were connected by wires that were tensioned prior to casting the web.



*Figure 3-1 Bridge over the river Marne at Luzancy designed by Freyssinet in 1941*

After Freyssinet's original work, the prestressing technique has been widely used in both Europe and North America by increasing its use over the decades in the design of bridges. As reported in [2], since the late '60s and '70s most medium-span bridges (30-90 m) and many long-span bridges up to 305 m of length, were built with prestressed concrete worldwide. For example, since 1965, about 47% of all bridges built in Germany were of prestressed concrete [3]. Prestressed concrete began to gain traction in the United State in the '50s and by the '80s, it accounted for 50% of all bridge construction [4]. In Italy, the large part of the bridges stock was built between the '60s and '80s decades due to the Italian economic growth and infrastructure development and it is mainly composed by simply supported prestressed concrete beam-type, more than 90% [5].

Generally, the prestressing force can be applied through two methods. One method is the pre-tensioning in which the prestressing strands are tensioned between massive abutments in a casting yard prior to place the concrete in the beam form Figure 3-2 (a). After the concrete curing, the jacking pressure is released; in this way the prestress force is transferred to concrete by bond, mostly near the ends of the beam, and no special anchorage is needed. This procedure is employed in pre-casting plants. Differently, post-tensioning is a method in which the tendon is tensioned after the concrete curing by means of a mechanical anchors, as shown in Figure 3-2 (b). The prestressing steel is placed in ducts within the concrete section or external to the concrete section. The tendons are anchored against the concrete immediately after prestressing.



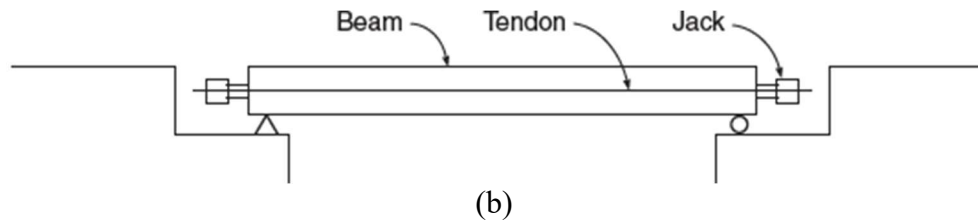


Figure 3-2 Prestressing methods (a) pre-tensioning, (b) post-tensioning [6]

The post-tensioning technique is the most vulnerable prestressing method and, it represents the one used in 40% of the total prestressed concrete bridges in Italy [5]. One of the concerns is related to the state of conservation of the precompression system that cannot be directly evaluated through visual inspection or conventional tests only. Therefore, in this thesis, the post-tensioning method is specifically analysed.

Following sections detail and present general principles for the design of a simple supported PT I-beam, the limit states, and the basic variables required to analyse the system response. The knowledge of the design process of a PT beam and of its basic random variables provides the base for simulating the assessment procedure of existing structures, topic that will be discussed in the Chapter 4.

## 3.2 GENERAL PRINCIPLES

In this section, the design process of a PT beam subjected only to vertical loads, is shown by focusing on *i*) pre-sizing rule; *ii*) the development of the cable and *iii*) the computation of prestress losses. Prescriptions are given by the recommendation in [7]-[10].

In the design of a prestressed concrete section, based on material properties and/or code prescriptions, the following parameters are known:

- the stress limits at the outer edges of the concrete section
- the stress limits for prestressing steel and mild reinforcement
- the nominal and design bending stresses acting on the section
- the self-weight of the prestressed element

The unknown variables often include:

- geometric parameters that define the optimal shape and dimensions of the prestressed section
- the location of the pressure centre from the prestressing cables within the specific section, effectively indicating the eccentricity pattern of the cables relative to the centroidal axis (of the homogenized ideal section)

- the extent of long-term stress losses due to time-dependent phenomena
- the degree of prestressing along the element
- the effects of any hyperstatic constraints or settlement on the prestressing and the overall structural alignment
- the viscous effects
- the extent of deformations, especially long-term ones due to the concrete's viscous effects

Among the unknown variables, the geometric ratios defining the section dimensions are particularly important, as these dimensions affect the magnitude of the prestressing forces required, which in turn impacts long-term effects like creep, shrinkage, relaxation, and thus deformations. Prestressed structures differ from reinforced concrete ones due to the predetermined shape of the section that it is heavily influenced by the initial conditions at the time of prestressing, and by the final service conditions with maximum variable load. In the initial design stage, the ratio  $\Omega$  between the variable and the total load is a factor that affects the resultant of compression and therefore the dimension of the beam as shown in Figure 3-3.

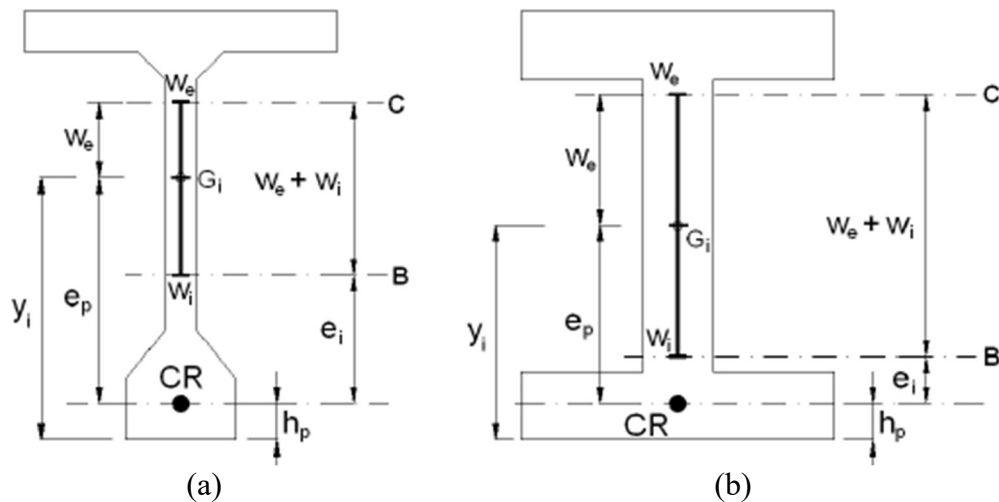


Figure 3-3 Variation of geometric dimension for (a) lower value and (b) higher value of  $\Omega$

Once that the optimal shape is chosen, some section proportions depend on factors unrelated to load magnitude that, however, directly impact the resistance design of the entire element. Fixed the span length  $L$ , the dimensions of a I-beam as the one showed in Figure 3-4 can be calculated using the following proportions [11]:

- $h = \frac{1}{15} \div \frac{1}{20} L$
- $b_i = 0.27 \div 0.35 h$
- $s_i = 0.10 h$

- $a_i = 0.12 \div 0.14 h$
- $s_e = 0.08 \div 0.12 h$
- $a_e = 0.05 \div 0.08 h$

The thickness  $h_t$  and width  $b_{eff}$  of the slab are function of the code and of the condition of use. More information are reported in [9].

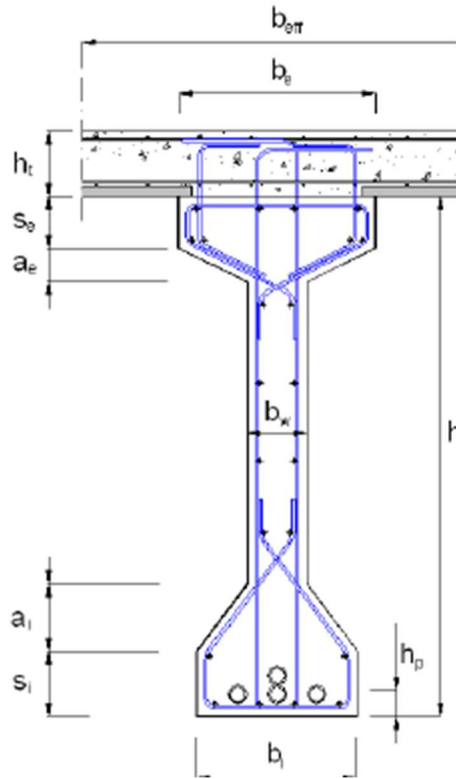


Figure 3-4 Dimensions of PT I-beam

The design of the prestressed beam is also influenced by the chosen level of precompression that defines the section. Three level of precompression, based on the maximum allowable tensile stress  $\sigma_{ct}$  during service condition, can be identified:

- full prestressing - no tensile stresses are allowed
- limited prestressing -  $\sigma_{ct} > 0$  is allowed at the outermost edge (without cracking)
- partial prestressing -  $\sigma_{ct} > 0$  and cracking permitted. This condition is unusable for bridges

Once chosen the section type, the initial prestressing force  $N_0$  and the final eccentricity  $e_p$  are evaluated for the most critical sections by only considering the case of simply supported beams that are the focus of this thesis. The nomenclature is introduced in the following:

- $M_{g0}$  is the nominal bending moment given by the self-weight
- $M_{g,ext}$  is the nominal bending moment due to all permanent external loads
- $M_Q$  is the bending moment due to the predominant variable loads (e.g. traffic)
- $N_0$  is the initial prestress force
- $N_{0,T}$  is the initial prestress force acting on the beam extremities
- $N_\infty$  is the long-term prestress force after all the prestressing losses
- $w_e$  is upper boundary of the central core of inertia of the section
- $w_i$  is lower boundary of the central core of inertia of the section
- $e_i$  distance from centre of gravity of the resulting cable (CR) to  $w_i$
- $e_p = w_i + e_i$  is the distance of the CR from the centre of gravity if the section
- $h_p$  is the distance from the CR from the bottom fiber of the beam
- $y_i$  is the distance from the bottom fiber to the centre of gravity of the beam section
- $J$  is the moment of inertia

In the case of full prestressing, the pre-dimensioning is obtained by solving the system of equations reported in Eq. (3-1) and that is based on the hypotheses that follow, and also depicted in Figure 3-5:

- $\sigma_{cte} = 0$ , null stress in the upper bound of the cross section in the initial phase (only self-weight  $G_0$ )
- $\sigma_{ct} = 0$ , null stress in the lower bound of the cross section in the final phase
- during the pre-dimensioning the final prestressing force  $N_\infty$  is assumed to be the 75% of the initial prestressing force  $N_0$

$$\begin{cases} e_i = \frac{M_{g0}}{N_0} \\ P_0 = (M_{g0} + (M_{g,est} + M_Q)) / (0.75 * (e_i + w_i + w_e)) \end{cases} \quad (3-1)$$

The initial phase corresponds to the cables stretching during which the reacting section is solely provided by the beam. In this stage, the beam is subjected to its self-weight, to the prestressing force and to the load given by the slab. At the final phase, also known as the operational condition, the reacting section is composed of both the beam and the slab. At this point, the acting loads are the permanent (self-weight and non-structural loads) and the traffic variable loads. Moreover, all cables have been stretched and injected.

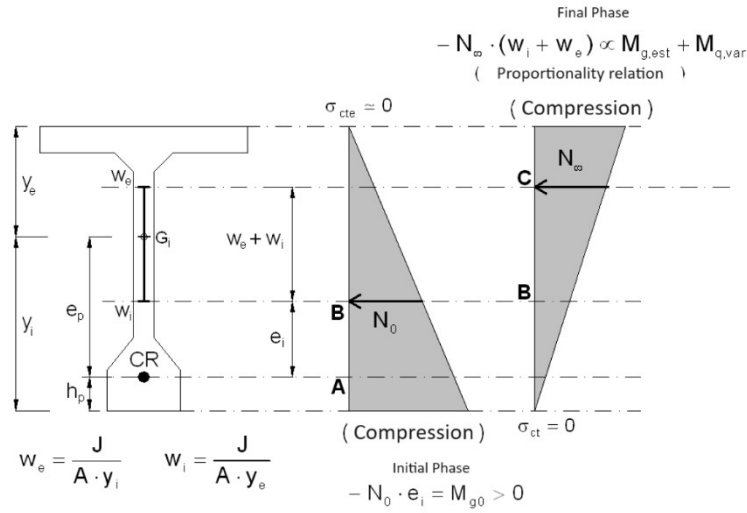


Figure 3-5 Stresses distribution in the case of full prestressing

In the case of limited prestressing, the pre-dimensioning is obtained by solving the system of equations reported as Eq. (3-2) that are based on the successive hypotheses, further shown in Figure 3-5:

- $0 < \sigma_{cte} < f_{ct}$ , tensile stress in the upper bound of the cross section in the initial phase (only self-weight  $G_0$ )
- tensile stress  $\sigma_{ct}$  in the lower bound of the cross section in the final phase equal to the allowable tensile
- during the pre-dimensioning the final prestressing force  $N_\infty$  is assumed to be the 75% of its starting force  $P_0$

$$\begin{cases} e_i = \frac{M_{g0}}{N_0} \\ P_0 = (M_{g0} + (M_{g,est} + M_q) - \sigma_{ct} \cdot J/y_i) / (0.75 * (e_i + w_i + w_e)) \end{cases} \quad (3-2)$$

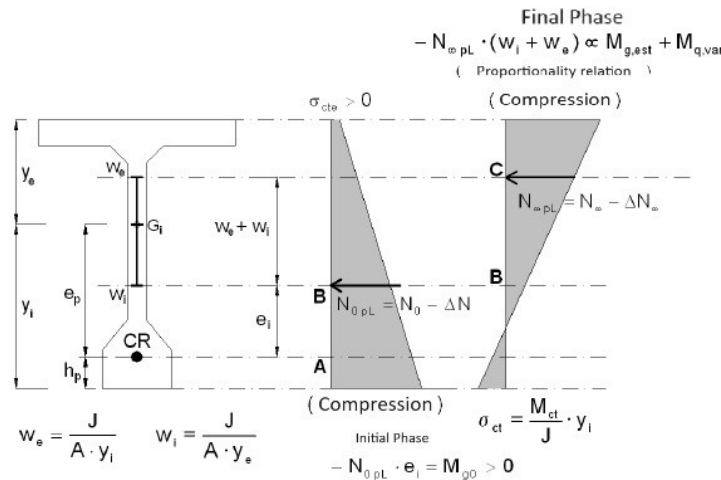


Figure 3-6 Stresses distribution in the case of limited prestressing

The total strand area is derived either for full and limited prestressing as the ratio between  $P_0$  and the maximum allowable stress on strand as specified in Section 3.3.1. Once defined the size of strands, their number is obtained and then distributed among the chosen number of cables.

After the pre-dimensioning, the most critical section is verified under serviceability limit state (SLS), as specified in Section 3.3.1. Preliminary, the short- and long-term prestress losses are estimated as a certain percentage of the initial force [11]. Once SLS checks are verified, the cables layout is determined. Aiming at ensuring that the tensile and compressive stresses remain within the Code's prescriptions, the centre of gravity of the equivalent cable should remain within a specific zone (identified as the grey shadowed one in Figure 3-7) all along the length of the beam. The lower bound of the specific zone is defined as the ratio between initial-phase moment and initial prestress force, while the upper bound is determined as the ratio between service-phase moments and final prestress force.

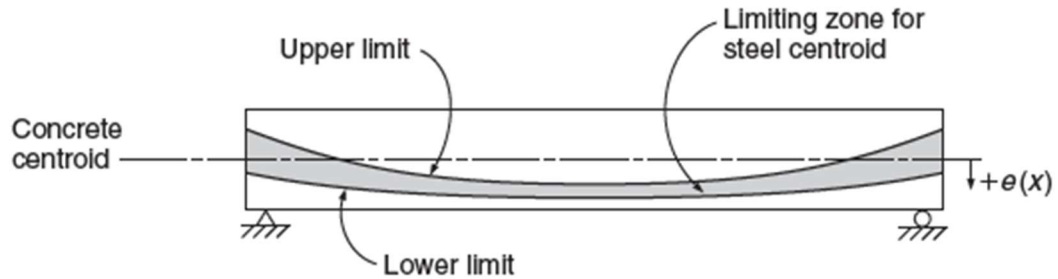


Figure 3-7 Limiting zone for steel cable development [6]

Once the path of cables is determined, the actual prestress losses are calculated following the Codes recommendations [9], [10], allowing for a re-evaluation of the section.

After completing this step, the safety checks of the section at the Ultimate Limit State (ULS) are performed as outlined in Section 3.3.2.

### 3.3 DESIGN ACCORDING TO LIMIT STATE

In this section, the design criteria of a simply supported post-tensioned concrete beam are introduced. These criteria are based on EC0 [7], EC1 [8], EC2 [9], and Italian Standard [10].

A bridge having an expected design service life of 100 years [7], must be designed and executed to ensure that:

- it can withstand all actions and environmental influences that can occur during construction and its service life

- it remains usable in safe conditions

The fulfilment of such requirements grants that the bridge has been designed having sufficient strength, serviceability and durability.

As described previously, a limit state is a condition defined by Standards beyond which the structure or part of the structure does no longer fulfil one of its performance requirements. Limit state is classified into two types: Ultimate Limit State (ULS) and Serviceability Limit State (SLS). SLS refers to the performance of the structure and its components during operational conditions of use, to ensure comfort to users and to visual aspects. At the ULS the bending and shear capacity of the structure is defined. Beyond this limit state the entire structure or part of it falls.

### 3.3.1 SERVICEABILITY LIMIT STATE (SLS)

The following SLS conditions must be satisfied:

- the initial stress on the strands must not be greater than the limit expressed in section 4.1.8.1.5 of the Italian Code [10] which is more restrictive with respect to the limit of section 5.10.2.1 of EC2 [9]

$$\sigma_{p0,max} = \min (0.75 * f_{ptk}, 0.85 * f_{p(0.1)k}) \quad (3-3)$$

- at the initial phase, the compression stress in the lower bound  $\sigma_{ic,0}$  of the section must be lower than  $0.7 f_{ck}$  as expressed in chapter 5.10.2.2 of EC2 [9] and in section 4.1.8.1.4 of [10]
- during the serviceability conditions (final phase), the compression strength in the upper bound of the section must be lower than:

$$\sigma_{c,max} < 0.60 f_{ck} \text{ for Characteristic Combination} \quad (3-4)$$

$$\sigma_{c,max} < 0.45 f_{ck} \text{ for Quasi-permanent Combination} \quad (3-5)$$

- during the serviceability condition (final phase), the stress on the cable cannot be greater than  $0.6 * f_{p(0.1)k}$
- the Decompression Limit State of the section is satisfied

The load combinations used in SLS, as specified in [8] and [10], are the Characteristic and Quasi-permanent combinations as defined in Eq. (3-6) and Eq. (3-7), respectively.

$$G + P + Q_{ki} \cdot \Psi_{0i} \text{ Characteristic Combination} \quad (3-6)$$

$$G + P + Q_{ki} \cdot \Psi_{2i} \text{ for Quasi-permanent Combination} \quad (3-7)$$

where  $G$  is the permanent load,  $P$  is the prestressing action,  $Q_{ki}$  is the  $i$ -th characteristic value of the variable load and  $\Psi_{i,k}$  is the  $i$ -th combination factor defined in [8] and [10].

### 3.3.2 ULTIMATE LIMIT STATE (ULS)

At the ULS, according to the partial factor method [7], [10] design values of resistance and action are determined and safety checks are carried out through the safety ratio  $\gamma_d$ . The partial factors  $\gamma$  are calibrated based on the structure's consequence class to ensure that the minimum  $P_f$  [7] is met. In the case of PT bridges, both the failure conditions related to shear and bending moment are considered since they are significantly influenced by the precompression system.

The shear capacity must be checked on both cracked and un-cracked section evaluated either in the case of unnecessary and necessary shear reinforcement. In the following all the possible conditions are presented.

In the case of cracked sections due to the bending moment, the shear capacity is derived using the following expressions:

$$V_{Rd,1} = \min(V_{Rsd}, V_{Rcd}) \quad (3-8)$$

$$V_{Rsd} = 0.9 \cdot d \cdot \frac{A_{sw}}{s} \cdot f_{ysd} \cdot (\cot \delta + \cot \vartheta) \cdot \sin \delta \quad (3-9)$$

$$V_{Rcd} = 0.9d \cdot b_w \cdot \alpha_c \cdot v \cdot f_{cd} \cdot (\cot \delta + \cot \vartheta) / (1 + \cot^2 \vartheta) \quad (3-10)$$

where  $d$  is the effective height of the cross-section;  $b_w$  is the minimum width between tension and compression chord;  $\vartheta$  is the angle between the concrete compression strut and the beam axis perpendicular to the shear action;  $\delta$  is the angle between shear reinforcement and the beam axis perpendicular to the shear force;  $A_{sw}$  is the cross-sectional area of the shear reinforcement;  $s$  is the spacing of the stirrups;  $f_{ysd}$  is the design value of the rebar yielding strength obtained as  $f_{ysk}/\gamma_s$ , where  $\gamma_s$  is a partial factor equal to 1.15;  $\alpha_c$  is the coefficient taking into account the state of stress in the compression chord;  $f_{cd}$  is the design value of the concrete strength obtained as  $f_{ck}/\gamma_c$ , where  $\gamma_c$  is a partial factor equal to 1.5; and  $v$  is the strength reduction factor for concrete cracked in shear. Further information can be found in sections 4.1.2.3.5.2 of [10] and 6.2.2 of [9]. However, although the codes require a minimum shear reinforcement, it may not always necessary. In such cases, the section can be considered unreinforced and its strength, governed by the concrete crushing, can be calculated using the following equation:

$$V_{Rd,2} = \max \left\{ \left[ C_{Rd,c} \cdot k \cdot (100 \cdot \rho_1 \cdot f_{ck})^{\frac{1}{3}} + k_1 \cdot \sigma_{cp} \right] \cdot b_w \cdot d \right. \\ \left. (v_{min} + 0.15\sigma_{cp}) \cdot b_w \cdot d \right\} \quad (3-11)$$

where  $d$  is the effective height of the cross-section;  $b_w$  is the smallest width of the cross-section in tensile area;  $k$  is a parameter depending on  $d$ ;  $f_{ck}$  is the characteristic value of the concrete compressive strength;  $C_{Rd,c}$  and  $k_1$  are parameters whose values are defined in national annexes, however their recommended values are  $0.18/\gamma_c$  and  $0.15$ , respectively;  $v_{min}$  is a parameter that depends on  $f_{ck}$ , and  $k$ ;  $\rho_1$  is a ratio depending on the area of tensile reinforcement,  $b_w$ , and  $d$ ;  $\sigma_{cp}$  is the axial stress in the cross section defined as the ratio between the actual prestressing force  $P_{pt,m}$ , and the area of concrete  $A_c$  of the cross section. Further information can be found in sections 4.1.2.3.5.1 of [10] and 6.2.2 of [9].

In the case of uncracked sections due to the bending moment, the shear capacity is derived as shown in Eq. (3-8)-(3-11). When shear reinforcement is not needed and the strength, governed can be calculated as:

$$V_{Rd,3} = 0.7 \cdot b_w \cdot d \cdot (f_{ctd}^2 + \sigma_{cp} \cdot f_{ctd})^{1/2} \quad (3-12)$$

where  $f_{ctd}$  is the design value of concrete tension strength evaluated as  $f_{ctk}/\gamma_c$ . This equation can be used in case in which the tensile tensions are lower than  $f_{ctd}$ . On the contrary, shear reinforcement is necessary.

In the case of post-tensioned prestressed elements, the shear contribution given by the vertical component of the prestressing force,  $V_{cp}$ , shall be added to obtain the total shear capacity,  $V_{Rd}$ . This component can be computed as

$$V_{cp} = P_{pt,m} \sin \alpha_{cp} \quad (3-13)$$

where  $P_{pt,m}$  is the prestressing force of the cross-section; and  $\alpha_{cp}$  is the angle between the prestressing force and the longitudinal axes of the beam. In the shear check the resultant cable is adopted for the evaluation of the prestressing force and of its location.

For both the cases of uncracked and cracked section, the safety shear ratios  $y_{d,s}$  are calculated as shown in Eq. (3-14) and Eq. (3-15) respectively.

$$y_{d,s} = \max \left( \frac{V_{cp} + V_{Rd,1}}{V_{ed}}, \frac{V_{cp} + V_{Rd,2}}{V_{ed}} \right) \quad (3-14)$$

$$y_{d,s} = \max \left( \frac{V_{cp} + V_{Rd,1}}{V_{ed}}, \frac{V_{cp} + V_{Rd,3}}{V_{ed}} \right) \quad (3-15)$$

$V_{ed}$  represents the shear action derived from the combination of loads at the ULS, as shown in Eq. (3-16):

$$y_{Gi} \cdot G_i + P + y_{Q1} \cdot Q_{ki} + y_{Qi} \cdot Q_{ki} \cdot \Psi_{0i}, \quad (3-16)$$

where  $G_i$  and  $Q_{ki}$  are the  $i$ -th permanent and characteristic loads respectively,  $y_{Gi}$  is the partial factor associated with the  $i$ -th permanent load and  $y_{Qi}$  is the partial factor associated with the  $i$ -th variable load.

In the case of bending check over a generic cross section, the resistance bending moment  $M_{Rd}$ , can be computed as

$$M_{Rd} = T d^* \quad (3-17)$$

$$T = f_{ypd} A_{pm} + f_{ysd} A_s \quad (3-18)$$

where  $T$  is the resultant tensile force containing the contributions given by the resultant cable, and by the reinforcement steel located under the neutral axis at the ultimate condition;  $f_{ypd}$  is the design value of the yielding strength of prestressing steel obtained as  $f_{ypk}/\gamma_s$ ;  $A_s$  is the area of the reinforcing steel;  $d^*$  is the arm between the two resultant compressive and tensile forces. The bending check is usually carried out at the midpoint of the simply supported beam since that section is the most stressed one.

The safety bending ratio is obtained as

$$y_{d,b} = \frac{M_{Rd}}{M_{ed}} \quad (3-19)$$

where  $M_{ed}$  is the design bending moment due to external actions.

Previous Eqs. (3-10)-(3-13) and (3-17) are sensitive to the characteristics of the post-tensioning system that in turn involves the area of the cables  $A_{pm}$ , the actual prestressing force  $P_{pt,m}$ , and the characteristic yield stress of the harmonic steel  $f_{ypk}$ . The resistance bending moment is clearly sensitive to  $A_{pm}$  and  $f_{ypk}$ , while the shear resistance is, instead, sensitive to the prestressing force. The  $P_{pt,m}$  influences Eq. (3-10) through the term  $\alpha_c$  representing the compressive state of the section given by the prestressing system, while  $P_{pt,m}$  explicitly appears in Eqs. (3-11)-(3-13). Moreover, all the previous parameters indirectly influence the shear capacity because they influence the location of the first cracked section in which  $V_{cp}$  is different. By reducing the prestressing action, the first cracked section moves towards the beam support, resulting in a higher  $V_{ed}$ .

### 3.4 BASIC RANDOM VARIABLES

The theoretical model describing the response (or capacity) of a system is defined by a set of basic variables  $\mathbf{x}$ , which represent the minimum number of

parameters required to analyse the system's response and the effect of loads. The system response includes information about geometrical, mechanical and material properties. Bridge assessment consists in evaluating whether its structural capacity is higher or at least equal to the effects of actions (demand), underlining that both are affected by several uncertainties. Action is subdivided in the effects given by permanent and live loads. In this section, the probabilistic model of live loads is discussed only for the traffic one; the effect of other loads (e.g. wind, snow, temperature, etc) are not addressed in this thesis.

In the case of PT beams, the vector of basic variables  $\mathbf{x}$  is defined as follows

$$\mathbf{x} = [A_p, P_{pt}, f_{up}, f_{ys}, f_c, G, Q] \quad (3-20)$$

where  $A_p$  denotes the wires area;  $P_{pt}$  is the current prestressing force;  $f_{up}$  is the ultimate strength of prestressing steel;  $f_{ys}$  is the yielding strength of the rebar;  $f_c$  is the concrete strength;  $G$  is the permanent load and  $Q$  is the the traffic load. These basic variables are random, and, in this section, their probabilistic models are provided. The geometric uncertainty related to deviations in the shape and dimensions of the cross-section is considered negligible in this thesis.

### 3.4.1 MATERIAL PROPERTIES

#### 3.4.1.1 CONCRETE

Concrete is composite material consisting of cement past and aggregates. Being a heterogeneous material it is characterized by non-linear and rheological behaviour; therefore, its mechanical behaviour is defined by models describing separately the short-term and long-term loading. The first describes the standard stress-strain relationship, while the second describes the development of stresses over time.

Under short-term quasi-static loading the mechanical behaviour of concrete can be described by the schematic stress-strain relationship shown in Figure 3-8.

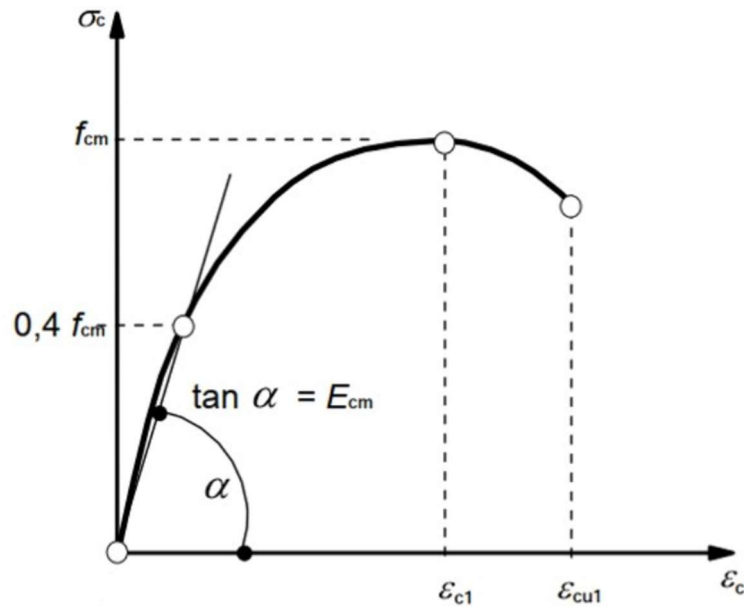


Figure 3-8 Concrete stress-strain relationship

The variability of the mechanical properties of concrete depends mostly on the following factors:

- properties of members material (cement, aggregate, etc.)
- concrete composition (w\c ratio, cement quality, etc.)
- execution (mixing, transporting, placing, curing and hardening, etc.)
- testing procedure (type and dimensions of specimens, load application, etc.)
- concrete pulled out from an existing structure
- maintenance, material degradation, etc

The reference and main investigated property of concrete is the compressive strength  $f_c$ . Other mechanical properties are  $f_{ct}$  (tensile strength),  $E_c$  elasticity modulus,  $\epsilon_{cu}$  that are usually defined through empirical relations whit  $f_c$ .

In the following, probabilistic models of concrete properties from codes and literature are reported and resumed.

According to JCSS [12], the concrete strength at a particular point  $i$  in a given structure  $j$  as a function of standard strength  $f_{c0}$ , derived from standard test specimens, is given as

$$f_{c,ij} = \alpha(t, \tau)(f_{c0,ij})^\lambda Y_{1,j} \quad (3-21)$$

$$f_{c0,ij} = \exp(U_{ij}\sigma_j + \mu_j) \quad (3-22)$$

where:

- $f_{c0,ij}$  is the log-normal variable, independent of  $Y_{1,j}$ , with distribution parameters  $\mu_j$  and  $\sigma_j$
- $\mu_j$  is the logarithmic mean
- $\sigma_j$  is the logarithmic standard deviation
- $Y_{1,j}$  is a lognormal variable representing additional variations due to placing, curing, and hardening conditions of in-situ concrete
- $U_{ij}$  is a standard normal variable representing variability within one structure
- $\lambda$  is a lognormal variable with  $\mu$  equal to 0.96 and  $c_v$  equal to 0.005, generally considered as deterministic parameters

The other mechanical properties as  $f_{ct}$ ,  $E_c$ ,  $\epsilon_{cu}$  are given in the following Eqs.

$$f_{ct,ij} = 0.3 f_{c,ij}^{2/3} Y_{2,j} \quad (3-23)$$

$$E_{c,ij} = 10.5 f_{c,ij}^{1/3} Y_{3,j} (1 + \beta_d \varphi(t, \tau))^{-1} \quad (3-24)$$

$$\epsilon_{cu,ij} = 6 \cdot 10^{-3} f_{c,ij}^{-1/6} Y_{4,j} (1 + \beta_d \varphi(t, \tau)) \quad (3-25)$$

where  $\beta_d$  is the ratio of permanent load to the total load;  $\varphi(t, \tau)$  is the creep coefficient; the variables  $Y_{2,j}$  to  $Y_{4,j}$  reflect variation due to factors not accounted for by concrete compressive strength like gravel type and size, chemical conditions and so on. The variables  $Y_{k,j}$  can be modelled according to log-normal distribution with parameters collected in Table 3-1 if direct measurements are not available.

Table 3-1 Data Parameters  $Y_{1,j}$

| Parameter          | Variable  | Distr. type | $\mu$ | $c_v$ |
|--------------------|-----------|-------------|-------|-------|
| Compression        | $Y_{1,j}$ | Lognormal   | 1     | 0.06  |
| Tension            | $Y_{2,j}$ | Lognormal   | 1     | 0.3   |
| Elasticity modulus | $Y_{3,j}$ | Lognormal   | 1     | 0.15  |
| Ultimate strain    | $Y_{4,j}$ | Lognormal   | 1     | 0.15  |

Besides the JCSS code model, several studies, derived from experimental campaign, are presented in the literature, which provide the distribution parameters of concrete strength for site and plant cast concrete and for several concrete classes. The statistical parameters of site-cast concrete are given in Table 3-2 and in Table 3-3. The statistical properties are reported related to the nominal value of characteristic concrete strength  $f_{ck}$ , while the bias  $\lambda_c$  is the ratio between the mean and characteristic value.

*Table 3-2 Site-cast concrete*

| Origin<br>(Reference)         | Nominal<br>value<br>$f_{ck}$ [MPa] | Bias<br>$\lambda_c$ | Standard dev.<br>$\sigma_c$ | Coef. of Var.<br>$c_v$ |
|-------------------------------|------------------------------------|---------------------|-----------------------------|------------------------|
| US, Canada,<br>Europe<br>[13] | $< 27$                             | —                   | —                           | 10 – 20                |
|                               | $\geq 27$                          | —                   | 2.7 – 5.4                   | —                      |
| Sweden<br>[14]                | 35                                 | 1.24                | —                           | 8.5                    |
| Germany<br>[15]               | 25 – 45                            | —                   | —                           | 9 – 20                 |
| Spain<br>[15]                 | 25 – 45                            | 1.09 – 1.39         | 2.7 – 5.4                   | 6 – 11                 |
| Canada<br>[16]                | $\leq 55$                          | 1.25                | —                           | 10                     |
| Austria<br>[17]               | 25 – 50                            | 1.02 – 2.04         | 1 – 2.7                     | 2 – 6                  |
| US<br>[18]                    | 21 – 41                            | 1.12 – 1.35         | 1.5 – 4.9                   | 4 – 15                 |
|                               | 48 – 83                            | 1.04 – 1.19         | 5.4 – 9.0                   | 9 – 12                 |

*Table 3-3 Plant-cast concrete*

| Origin<br>(Reference) | Nominal<br>value<br>$f_{ck}$ [MPa] | Bias<br>$\lambda_c$ | Standard dev.<br>$\sigma_c$ | Coef. of Var.<br>$c_v$ |
|-----------------------|------------------------------------|---------------------|-----------------------------|------------------------|
| Germany<br>[15]       | 50 – 65                            | —                   | —                           | 6                      |
| Canada<br>[16]        | $\leq 55$                          | 1.19                | —                           | 5                      |
| Europe<br>[19]        | 40 – 90                            | —                   | 2 – 6                       | 3 – 11                 |
| US<br>[18]            | 34 – 45                            | 1.14 – 1.38         | 4.1 – 5.7                   | 8 – 12                 |

By knowing the  $f_c$ , the other concrete properties can be derived through empirical approaches.

For the tensile strength [20] suggests to model as lognormal distribution having a mean value equal to Eq. (3-26) and with the  $c_v$  ranging between 0.18-0.20.

$$f_{ctm} = 0.30f_{ck}^{2/3} \quad (3-26)$$

For concretes having strength greater than 50/60, the Eurocodes 2 [9] proposes the following relation:

$$f_{ctm} = 2.12 \ln \left( 1 + \frac{f_{cm}}{10} \right) \quad (3-27)$$

The secant elasticity modulus can be modelled as normal or lognormal distributed with a mean value, according to [9], defined by the Eq. (3-28) and a  $c_v$  of 0.12 [13].

$$E_{cm} = 22000 \left( \frac{f_{cm}}{10} \right)^{0.3} \quad (3-28)$$

#### 3.4.1.2 REINFORCING STEEL

The mechanical behaviour of reinforcing steel is primarily characterized by the ultimate tensile strength, yield strength, and ductility or deformation capacity. By knowing these parameters, the behaviour of reinforcing steel can be accurately defined through stress-strain curves as the one depicted in Figure 3-9.

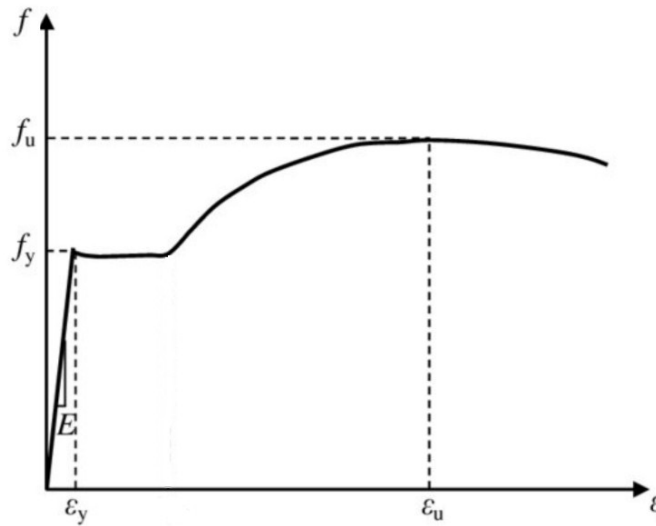


Figure 3-9 Steel stress-strain relationship

The variability of the mechanical properties of reinforcing steel is generally lower compared to that of concrete parameters, primarily due to the higher level of industrialization in the production process and stricter quality control. The main

factors influencing the variability of the mechanical behaviour of reinforcing steel bars are:

- variability in material strength
- variability in bar geometry
- material degradation
- load history
- experimental evaluation

Several mechanical properties of reinforcing steel can be determined through experimental testing, such as yield strength  $f_{sy}$ , ultimate strength  $f_{su}$ , ultimate strain  $\epsilon_{su}$ , etc. However, the most frequently tested property is yielding strength. In the following sections, the probabilistic models of strength and geometric properties of reinforcing steel, as found in specialized codes and technical literature, are summarized.

According to JCSS [12], the statistical parameters of reinforcing steel are summarized in Table 3-4. For the showed parameters a normal or lognormal distribution can be adopted.

*Table 3-4 Statistical parameters of reinforcing steel properties*

| Steel Property    | Mean value<br>$X_m$    | Standard dev.<br>$\sigma_X$ | Coef. of Var.<br>$c_v$ |
|-------------------|------------------------|-----------------------------|------------------------|
| Bar Area          | $A_{s,nom}$            | —                           | 0.02                   |
| Yield strength    | $f_{s,nom} + 2 \sigma$ | 30 MPa                      | —                      |
| Ultimate strength | —                      | 40 MPa                      | —                      |
| Ultimate strain   | —                      | —                           | 0.09                   |

Besides the JCSS code model, several models provide distribution parameters of  $f_{sy}$ . These are based on tests carried out on steel produced over the decades and from different part of the world (US, Canada, Europe, etc.). These results are reported in Table 3-5.

*Table 3-5 Experimental results of yielding strength*

| Origin<br>(Reference) | Nominal<br>value<br>$f_{syk}$ [MPa] | Bias<br>$\lambda_{sy}$ | Coef. of Var.<br>$c_v$ |
|-----------------------|-------------------------------------|------------------------|------------------------|
|-----------------------|-------------------------------------|------------------------|------------------------|

|                               |            |              |             |
|-------------------------------|------------|--------------|-------------|
| Denmark<br>[15]               | 370 – 420  | –            | 4; 10       |
| US, Canada,<br>Europe<br>[13] | 280<br>410 | 1.20<br>1.20 | 10.7<br>9.3 |
| Spain<br>[15]                 | 500        | 1.20         | 8.1         |
| Europe<br>[17]                | 500        | 1.156        | 2.3         |
| US<br>[18]                    | 420        | 1.145        | 5           |

In Italy, a detailed study [22] has been conducted to determine the statistical parameters of yield strength of different steels category mainly used for building between the '50s and '80s. These results are summarised in Table 3-6. For the showed parameters a normal or lognormal distribution can be adopted.

*Table 3-6 Statistical Parameters Italian steel category between 1950s and 1980s*

| Steel<br>Category | Mean value<br>$f_{ym}$ [MPa] | Standard dev.<br>$\sigma_y$ [MPa] | Coef. Of Var.<br>$c_v$ |
|-------------------|------------------------------|-----------------------------------|------------------------|
| Common<br>steel   | 276.47                       | 25.10                             | 0.09                   |
| AQ42              | 322.34                       | 26.59                             | 0.08                   |
| AQ50              | 370.9                        | 31.84                             | 0.085                  |
| AQ60              | 435.45                       | 50.06                             | 0.11                   |
| ALE               | 512.06                       | 65.41                             | 0.13                   |
| FeB22K            | 325.11                       | 39.83                             | 0.12                   |
| FeB32K            | 430.03                       | 53.48                             | 0.12                   |

### 3.4.1.3 PRESTRESSING STEEL

In concrete structures, three types of steel are usually used as prestressing tendons: cold drawn wires; strands and high-strength bars; all of them are described and regulated by EN10138 [23].

The mechanical behaviour of prestressing steel varies based on the following factors:

- variability in material strength
- variability in bar geometry
- material degradation
- load history
- experimental evaluation

A typical stress-strain diagram of prestressing steel is reported in Figure 3-10. As it can be seen, prestressing steel does not have a distinct yield point, so that the yield stress is determined according to strain criterion. Depending on the standard, yield stress,  $f_{py}$ , can be defined as the stress at the 0.1% offset,  $f_{py,0.1}$  (which can also referred to as 0.1% proof stress); or as the stress at the 0.2% offset,  $f_{py,0.2}$  (or 0.2% proof stress); or, according to ASTM standards, as the stress that corresponds to a total strain of 1% for wires and strands and of 0.75% for prestressing bars. Besides those parameters, other mechanical properties are the ultimate limit strength  $f_{pu}$ , the modulus of elasticity  $E_p$  and the ultimate strain  $\varepsilon_{pu}$ .

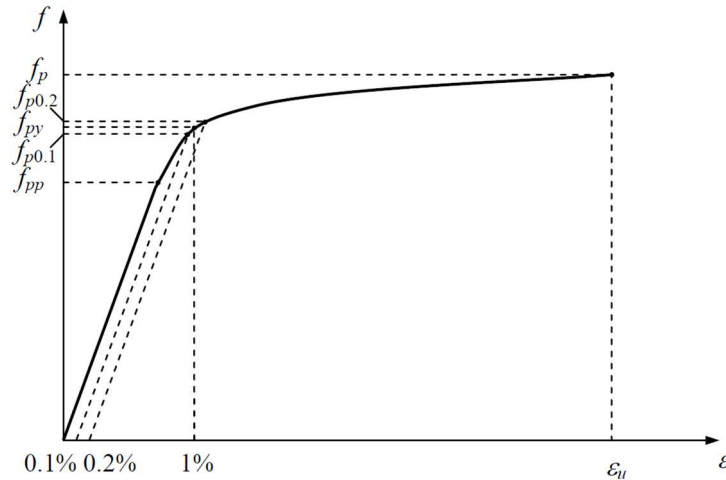


Figure 3-10 Prestressing steel stress-strain relationship

The probability distributions of the mechanical properties of prestressing steel can be modelled as Normal or Lognormal. According to JCSS [12], the essential mechanical properties are  $f_{pu}$ ,  $E_p$  and  $\varepsilon_{pu}$  and are presented in Table 3-7.

Table 3-7 Statistical properties of prestressing steel [12]

| Steel Property                        | Mean value<br>$X_m$ | Standard dev.<br>$\sigma_X$ | Coef. of Var.<br>$c_v$ |
|---------------------------------------|---------------------|-----------------------------|------------------------|
| Ultimate strength<br>$f_{pu}$         | $1.04f_{pk}$        | —                           | 0.025                  |
|                                       | $f_{pk} + 66$       | 40 MPa                      | —                      |
|                                       | 200 GPa for wires   | —                           | 0.02                   |
| Ultimate strength<br>$E_p$            | 195 GPa for strands | —                           | 0.02                   |
|                                       | 200 GPa for bars    | —                           | 0.02                   |
| Ultimate strain<br>$\varepsilon_{pu}$ | 0.05                | 0.0035                      | —                      |

By defining  $f_{pu}$  the other stresses can be computed adopting empirical correlations [9],[24] as shown in Table 3-8.

Table 3-8 Correlation between ultimate strength and other stresses

| Steel Type | $f_{pp}$     | $f_{py0.1}$  | $f_{py0.2}$  | $f_{py1}$    |
|------------|--------------|--------------|--------------|--------------|
| Wire       | $0.70f_{pu}$ | $0.86f_{pu}$ | $0.90f_{pu}$ | $0.85f_{pu}$ |
| Strand     | $0.65f_{pu}$ | $0.85f_{pu}$ | $0.90f_{pu}$ | $0.85f_{pu}$ |
| Bar        | —            | —            | —            | $0.85f_{pu}$ |

Besides the JCSS code model, several models provide distribution parameters of  $f_{pu}$  and  $f_{py}$ . These are based on tests carried out on prestressing steel produced over the decades and from different part of the world (US, Canada, Europe, etc.). These results are reported in Table 3-9.

Table 3-9 Experimental results of prestressing steel

| Origin<br>(Reference) | Str | Nominal<br>characteristic<br>value [MPa] | Bias<br>$\lambda_{sy}$ | Coef. of Var.<br>$c_v$ |
|-----------------------|-----|------------------------------------------|------------------------|------------------------|
|-----------------------|-----|------------------------------------------|------------------------|------------------------|

|                     |          |      |             |           |
|---------------------|----------|------|-------------|-----------|
| US<br>[25]          | $f_{py}$ | 1670 | 1.06        | 1.3       |
|                     | $f_{pu}$ | 1860 | 1.02        | 1.1       |
| Spain<br>[15]       | $f_{py}$ | 1760 | 1.04 – 1.06 | 1.7 – 2.5 |
|                     | $f_{pu}$ | 1860 | 1.04 – 1.06 | 1.8 – 2   |
| Europe<br>[17] [17] | $f_{py}$ | 1570 | 1.07 – 1.14 | 0.6 – 2.3 |
|                     | $f_{pu}$ | 1770 | 1.03 – 1.08 | 0.5 – 2.2 |
| US<br>[18]          | $f_{py}$ | 1720 | 1.07 – 1.14 | 1 – 3     |
|                     | $f_{pu}$ | 1860 | 1.04 – 1.06 | 1 – 3     |

Regarding the strand area, it can be modelled as log-normal distributed having a mean equal to nominal its value [7] and a coefficient of variation equal to 0.02 [23].

### 3.4.2 PRESTRESSING FORCE

The effect of prestressing on structural concrete is typically seen as a prestressing force applied to the concrete. This force  $P(x, t)$ , at a distance  $x$  and at the time  $t$ , can be expressed as

$$P(x, t) = P_0 - \Delta P(x, t) \quad (3-29)$$

where  $P_0$  represents the initial prestressing force, and  $\Delta P(x, t)$  are the prestress losses. These losses depend on various factors, including the type of prestressing, as well as the properties of the prestressing steel and concrete, and environmental conditions.  $\Delta P(x, t)$  is given by the sum of immediate  $\Delta P(x, t_0)$  and long term  $\Delta P(x, \infty)$  losses, that can be estimated using different design codes as previously shown.

$P_0$  can be treated as Normal or Lognormal distributed variable, whose mean value can be derived from design. The prestress losses too can be model as Normal or Lognormal variable. Their coefficients of variation are shown in Table 3-10.

*Table 3-10 Coefficient of variation prestressing force and prestress force*

| Parameter                            | $c_v$               |                        |
|--------------------------------------|---------------------|------------------------|
|                                      | Immediate $t = t_0$ | Immediate $t = \infty$ |
| Prestress losses<br>$\Delta P(x, t)$ | 0.30                | 0.30                   |
| Prestressing force<br>$P(x, t)$      | 0.04 – 0.06         | 0.06 – 0.09            |

### 3.4.3 LOADS

The loads are actions that can be subdivided in a group of forces acting on a structure (direct action) and in a group of imposed deformations or accelerations (indirect action). Based on their variation in time, they can be classified as follows:

- permanent actions  $G$ : actions that are present during the entire lifetime of the structure. Their change over the time is negligible. In the case of concrete bridges, they are mainly the self-weight of structural elements and of non-structural elements like floor, road finishing, etc.
- variable actions  $Q$ : actions that are not present during the entire reference period. Their change over time is not negligible. With reference to concrete bridges the main variable action is the traffic load
- accidental actions  $A$ : actions that have a low probability of occurrence during the reference period, but can have important consequences in the structure (e.g. earthquakes, floods, ...)

Action can be also classified with respect to the structural response:

- static loads are those that do not produce significant accelerations in the structural system or in its components
- dynamic loads are those that can produce significant accelerations in the structural system or in its components. This type of actions is treated in chapters 6 and 7.

Loads must be considered as random variables with a certain distribution type. In the following section, the distribution models characterizing the permanents and variable loads are showed. However, considering variable loads as stochastic processes is sometimes more appropriate.

### 3.4.3.1 PERMANENT

For bridges, the mainly permanent loads include the weight of the structural elements (bridge deck), and non-structural elements (such as road paving, shoulders, etc.). The uncertainty of permanent loads arises mainly from variations in material density, as well as the dimensional variation, which have been previously discussed. If dimensional variations are disregarded, material density becomes the key factor contributing to this uncertainty. However, the variability of permanent loads is typically small and can be often neglected [8], in this case the characteristic value is equal to the nominal value. When these loads are modelled as random variables, normal and lognormal distribution are commonly used [12].

The statistic parameters of density are proposed by JCSS [12] and resumed in Table 3-11. The weight density can be assumed to be normal or lognormal distributed

*Table 3-11 Statistical parameters material density*

| Material               | Mean Value<br>$\mu$ [ $kN/m^3$ ] | Coef. of Var.<br>$c_v$ |
|------------------------|----------------------------------|------------------------|
| Steel                  | 77                               | < 0.01                 |
| Ordinary concrete      | 24                               | 0.04                   |
| High strength concrete | 24 – 26                          | 0.03                   |
| Light concrete         | Depend on mix                    | 0.04 – 0.08            |
| Cellular concrete      | Depend on mix                    | 0.05 – 0.10            |

Eurocode 1 [8] suggests to consider for permanent loads a coefficient of variation between 0.05 and 0.10 depending on the type of structure.

In the case of studies based on experimental campaigns, [27] recommends a coefficient of variation for factory and cast-on site elements equal to 0.08 and 0.10 respectively.

### 3.4.3.2 VARIABLE LOADS

Several variable loads can act on a bridge, including vehicular and pedestrian traffic, temperature, wind, snow, and more. However, this thesis focuses solely on traffic loads, as they have the most significant impact on concrete road bridges.

The effect of traffic loads depends on several function, such as bridge span, vehicle weight, axles configuration, position of the vehicle on the bridge, number of vehicles, number of lanes. Given the variability of many of these factors over time, modelling the effect of traffic loads is a highly complex task. In standard bridge design and assessment practices, this complexity is typically addressed by using load models defined in engineering codes.

The traffic load models proposed in Eurocode 1 [8] consist of load models composed by combinations of a number of concentrated loads, representing heavy axles trucks, and a distributed load representing light vehicles. These loads are specified by their characteristic value  $Q_k$ , which for a variable load is generally determined as the value that corresponds to an over exceedance probability  $p$  during a reference period  $t_{ref}$ . The variable load can also be related to a return period  $T_R$ , which represent the time between the subsequent occurrences of an event. By fixing  $p$  and  $t_{ref}$ ,  $T_R$  can be obtained as shown in Eq. (3-30)b . Figure 3-11 shows the  $p$  and  $t_{ref}$  and, consequently,  $T_R$  for the several load models proposed in [8].

$$T_R = \frac{1}{1 - (1 - p)^{1/t_{ref}}} \quad (3-30)$$

The random nature of variable loads is modelled using a Gumbel distribution (Appendix A). As shown in Figure 3-11, the reference period used is typically 50 years. However, for reliability analysis, it is more appropriate to adopt a  $t_{ref}$  of 1 year to estimate the annual failure probability. The transition from  $t_{ref} = 50$  years to  $t_{ref} = 1$  year can be made using Eq. (3-31) and (3-32).

$$\mu_{50} = \mu_1 + \frac{\ln(50)}{a} \quad (3-31)$$

$$\sigma_{50} = \sigma_1 \cong \frac{1.282}{a} \quad (3-32)$$

where  $\mu_{50}$  and  $\mu_1$  are the mean value of the distribution of  $t_{ref} = 50$  years and = 1 year, respectively, while  $\sigma_{50}$  and  $\sigma_1$  are the distributions standard deviations. The parameter  $a$  is the scale factor of Gumbel distribution. The mean and the standard deviation for both reference periods are reported in Table 3-12.

| Traffic Load Models | Characteristic values                                                                                                                                       | Frequent values                                                                                         | Quasi-permanent values                                      |
|---------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------|-------------------------------------------------------------|
| <b>Road bridges</b> |                                                                                                                                                             |                                                                                                         |                                                             |
| LM1<br>(4.3.2)      | 1000 year return period (or probability of exceedance of 5% in 50 years) for traffic on the main roads in Europe ( $\alpha$ factors equal to 1, see 4.3.2). | 1 week return period for traffic on the main roads in Europe ( $\alpha$ factors equal to 1, see 4.3.2). | Calibration in accordance with definition given in EN 1990. |
| LM2<br>(4.3.3)      | 1000 year return period (or probability of exceedance of 5% in 50 years) for traffic on the main roads in Europe ( $\beta$ factor equal to 1, see 4.3.3).   | 1 week return period for traffic on the main roads in Europe ( $\beta$ factor equal to 1, see 4.3.3).   | Not relevant                                                |
| LM3<br>(4.3.4)      | Set of nominal values. Basic values defined in annex A are derived from a synthesis based on various national regulations.                                  | Not relevant                                                                                            | Not relevant                                                |
| LM4<br>(4.3.5)      | Nominal value deemed to represent the effects of a crowd. Defined with reference to existing national standards.                                            | Not relevant                                                                                            | Not relevant                                                |

Figure 3-11 Return period and probability of over exceedance for traffic load models [8]

Table 3-12 Statistical characteristic of time-variant load traffic [28]

| Variable     | Distr. | $t_{ref}$ | Bias ( $\mu_X/X_k$ ) | Coef. of Var.<br>$c_v$ |
|--------------|--------|-----------|----------------------|------------------------|
| Traffic load | Gumbel | 1 year    | 0.73                 | 0.075                  |
| Traffic load | Gumbel | 50 years  | 0.90                 | 0.061                  |

### 3.5 REFERENCES

- [1]Billington, D.P., 2004. Historical Perspective on Prestressed Concrete. PCI JOURNAL.
- [2]Lin, T.Y., Burns, N.H., 1981. Design of Prestressed Concrete Structures. Johan Wiley & Sons
- [3]Lin, T.Y., Fulka, F., 1975. Fifty-Year Advancement in Concrete Bridge Construction. J. Const. Div., Am. Soc. Of Civil Engineers.
- [4]Shri, B., Culmo, M., Ma, J., Martin, B., Russel, H., 2006. The Interstate Highway System and the Development of Prestressed Concrete Bridges in 50 Years of Interstate Structures: Past, Present, and Future. Transportation Research Circular, No. E-C104
- [5]Pinto, P. E., Franchin, P., 2010. Issue in the Upgrade of Italian Highway Structures. Journal of Earthquake Engineering, 14:8, 1221 – 1252: <http://dx.doi.org/10.1080/13632461003649970>
- [6]Darwin, D., Dolan, C.W., Nilson, A.H., 2016. Design of Concrete Structures. Fifteenth edition. ed. McGraw-Hill Education, New York, NY.
- [7]CEN, 2002. EN 1990 Eurocode 0 - Basis of Structural Design, Brussel.
- [8]CEN, 2005. EN 1991 Eurocode 1 – Action on Structures. Brussels.
- [9]CEN 2005. EN 1992-1 Eurocode 2 – Design of Concrete Structures, Part 1. Brussels.
- [10]Italian Ministry of Infrastructures and Transport Decree 17/01/2018 n.42, 2018. Adjournment of Technical Code for Constructions. Rome, Italy
- [11]Sigmund, C. 2012. Calcolo di Elementi Prefabbricati in Cemento Armato Precompresso. Grafill, Italy (in Italian).
- [12]JCSS (2002) Probabilistic Model Code, Part 3: Resistance Models, Joint Committee on Structural Safety.
- [13]Mirza, S.A., Hatnikolas, M., M. & MacGregor, J.G., 1979. Statistical Descriptions of Strength of Concrete. Journal of the Structural Division, ASCE.
- [14]Thelandersson, S., 2004. Assessment of Material Property Data for Structural Analysis of Nuclear Containments – Rapport TVBK-3051. Lund University, Department of Structural Engineering.
- [15]Sobrino, J.A., 1993. Evaluation del Comportamiento Funcional t de la Seguridad Estructural de Puentes Existentes de Hormigòn Armado y Pretensado. Ph.D. thesis, Technical University of Catalonia, Department of Construction Engineering.
- [16]Barlett, F.M., MacGregor, J.G., 1996. Statistical Analysis of Compressive Strength of Concrete in Structures. ACI Material Journal, 93, 158-169.
- [17]Strauss, A., 2003. Stochastische Modellierung und Zuverlässigkeit von Betonkonstruktionen. Ph.D. thesis, University of Applied Science and Natural Resourced, Vienna, Department of Civil Engineering and Natural Hazard.
- [18]Nowak, A.S., Szerszen, M.M., 2003. Calibration of Design Code for Buildings (ACI 318): Part1 – Statistical Models for Resistance. ACI Structural Journal.
- [19]PCSF, 2002. Precast Concrete Safety Factors – Final Report. European Research, Contract SMT4 CT98 2267.
- [20]Spaethe, G., 1992. Die Sicherheit Tragender Baukonstruktionen, Zweite Neubearbeitete Auflage. Springer.

- [21] Eurocode, 2005. *EN 1992-1 Eurocode 2- Design of Concrete Structures*, Part1, European Committee for Standardization; Brussels, Belgium.
- [22] Verderame, G.M., Ricci, P., Esposito, M., Sansiviero, F.C., 1950. LE CARATTERISTICHE MECCANICHE DEGLI ACCIAI IMPIEGATI NELLE STRUTTURE IN C.A. REALIZZATE DAL 1950 AL 1980.
- [23] EN 10138. Prestressing Steels. CEN, Bruxelles.
- [24] CEB-FIB Model Code, 1990. CEB-FIP MC90. Bulletin D'Information No. 213/214, Lusanne, Switzerland.
- [25] Devalapura, R.K., Tadros, M.K., 1992. Critical Assesment of ACI 318 Eq. (18-3) for Prestressing Steel Stress at Ultimate Flexure. ACI Structural Journal.
- [26] CEN, 2005. EN 1991. Eurocode 1 – Actions on Structures. Brussels.
- [27] Nowak, A.S., Hong, Y., 1991. Bridge Live-Load Models. J. Struct. Eng. 117, 2757–2767. [https://doi.org/10.1061/\(ASCE\)0733-9445\(1991\)117:9\(2757\)](https://doi.org/10.1061/(ASCE)0733-9445(1991)117:9(2757))
- [28] Caspeelee, R., Van Den Hende, K., 2023. Validation of the harmonized partial factor method for design and assessment of concrete structures as proposed for *fib* model code 2020. Structural Concrete 24, 4368–4376. <https://doi.org/10.1002/suco.202201109>

# 4 RELIABILITY OF EXISTING POST-TENSIONED CONCRETE BRIDGES

## 4.1 INTRODUCTION

The reliability assessment of existing structures differs from the new ones in several aspects [1]:

- increased target reliability levels that usually involve huge increment of costs for existing structures rather than for the new structures
- shorter remaining working life
- limited or lacking information about the actual structural condition

Regarding the last point, the evaluation of existing structures suffers from the lack of knowledge about their current state, which is essential for the assessment. This gap can be partially addressed through visual inspections, experimental tests and measurements. However, in common practice, only a reduced and imprecise number of tests are typically carried out. This produces additional uncertainty due to the knowledge process. This uncertainty has considerable relevance and affects the evaluation [2].

This chapter focuses on the propagation of uncertainties of knowledge process, number of tests, confidence level and measurement error, that born any time that the reliability of existing structures is evaluated. Among all existing structures, the assessment of existing post-tensioned concrete bridges presents challenges, because the state of conservation of prestressing system cannot be inspected through visual inspection and conventional tests only. Moreover, while several studies in literature focus on simple concrete structures, no previous studies have examined this effect on PT members, which are characterized by a greater complexity, a higher number of basic variables, and potentially large uncertainties about measurement errors.

This chapter will first explore the evaluation of the existing structures adopting both the semi-probabilistic and the probabilistic approaches.

## 4.2 SEMI-PROBABILISTIC EVALUATION AND NOTATION

As introduced in Section 2.2, the reliability assessment is often performed using the semi-probabilistic approach [3]. This method derives a set of deterministic “design” parameters  $\boldsymbol{\vartheta}_d$ , from  $\mathbf{X}$ , vector of the basic random variables, and estimates through these parameters the safety ratio  $\gamma_d(\boldsymbol{\vartheta}_d)$ . In the case of new

structure, the characteristic values are known, and it is not necessary to know the statistical properties of  $f_X(\mathbf{x})$  of  $\mathbf{X}$ .

In the case of existing structures, such as those considered in this study, the  $f_X(\mathbf{x})$  of  $\mathbf{X}$  can be only estimated by a reduced number of imprecise tests. Consequently, the components of  $\boldsymbol{\vartheta}_d$  are no more deterministic values, but they are a r.v.  $\bar{\boldsymbol{\vartheta}}_d$ , that collect possible estimations  $\bar{\boldsymbol{\vartheta}}_d$  of  $\boldsymbol{\vartheta}_d$ . For example, the expected variability of  $\bar{\boldsymbol{\vartheta}}_d$  reduces by increasing the number of tests. Consequently, the expected safety ratio  $\bar{Y}_d = y_d(\bar{\boldsymbol{\vartheta}}_d)$  is a r.v. too.

Section 3.4 discussed the basic variables characterizing the strength of PT bridge beam and the acting loads. When adopting the semi probabilistic approach, the effect of knowledge is considered in the following elaborations as affecting only the strength of the PT beam. The loads are considered not affected by uncertainty and computed following the prescriptions given by [4]. With these assumptions, the parameters  $\boldsymbol{\vartheta}_d$  required for the safety checks (Section 3.3.2) are

$$\boldsymbol{\vartheta}_d = [A_{pm}, P_{pt,m}, f_{ysd}, f_{upd}, f_{cd}] \quad (4-1)$$

The vector  $\boldsymbol{\vartheta}_d$  encompasses the mean values of the wires area  $A_{pm}$ ; the mean value of the current prestressing force  $P_{pt,m}$ ; and the design values of the strength of materials  $f_{ysd}, f_{upd}, f_{cd}$  derived by the ratio between their characteristic value and the relevant partial factor.

As previously explained, existing structures requires that some design parameters  $\vartheta_d$  of  $\mathbf{X}$  are estimated from a limited number  $n$  of tests providing samples  $\bar{x}$ . Estimated parameters are r.v.  $\bar{\vartheta}_d$  and the confidence  $\alpha = P[\bar{\vartheta}_d < \vartheta_d]$  is the probability that the estimated parameter is lower than the actual value. If the Cumulative Density Function (CDF) of  $\bar{\vartheta}_d$  is available, it is possible to define the limit value  $\bar{\vartheta}_d^\alpha$  of the estimated quantity, such that  $F_{\bar{\vartheta}_d}(\bar{\vartheta}_d^\alpha) = \alpha$ .

Thus, the existing structure always deals with partial knowledge due to the limited number of tests. To simulate this scenario, starting from the probability distribution of  $f_X(\mathbf{x})$  of  $\mathbf{X}$  and fixing several tests  $n$ , multiple tests campaigns can lead to different estimates of the parameter distributions (with a confidence level  $\alpha$ ) and hence different values of the strength parameter  $\bar{\boldsymbol{\vartheta}}_d$ . From these, a probability distribution for the estimated safety ratio  $\bar{Y}_d$  is obtained. This distribution, showed in Figure 4-1, and its properties can be studied to evaluate the reliability of the method. It is interesting, indeed, to evaluate the probability that the estimated value  $\bar{y}_d$  is lower than the reference value  $y_d$ . This probability is equal to the value assumed by the CDF at the reference value,  $F_{\bar{Y}_d}(y_d)$  or, equivalently, the area under the function  $f_{\bar{Y}_d}(y_d)$  evaluated on the domain  $\bar{y}_d < y_d$  (Figure 4-1).

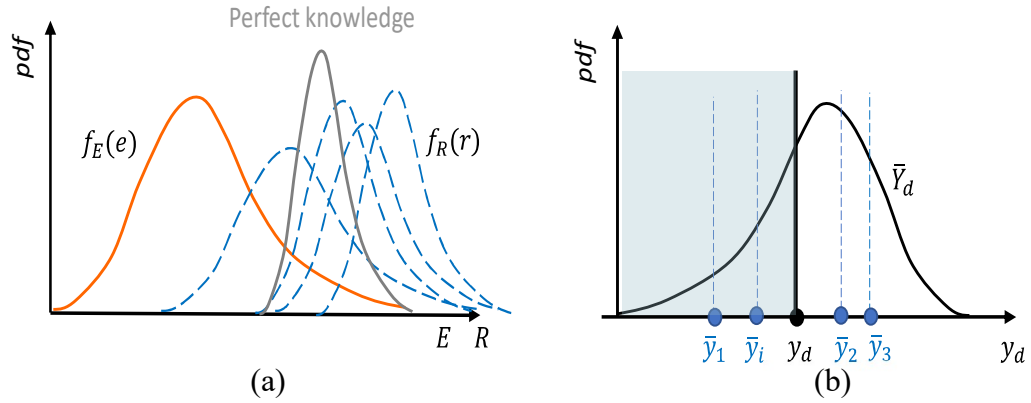


Figure 4-1 Safety assessment based on a partial knowledge: (a) strength distribution  $f_R(\mathbf{r})$  in the perfect knowledge scenario (solid grey line) and possible distributions (dashed blue line) derived from partial knowledge; (b) distribution of the estimated safety ratio  $\bar{Y}_d$  in which  $\bar{y}_i$  and  $y_d$  are, respectively, possible estimated values and the reference one.

#### ESTIMATION OF THE MEAN VALUE

The mean values of the wires area ( $A_{pm}$ ) and of the prestressing force ( $P_{pt\_m}$ ), derived from samples are estimated using the recommendation in Appendix A for known and unknown population variance.

#### ESTIMATION OF THE FRACTILE WITH CONFIDENCE

If a parameter of a distribution can be estimated from samples  $n$  with a defined confidence level  $\alpha$ , the value at a given fractile  $p$  of a distribution can also be estimated. In this case, the characteristic values  $f_k$  of the material strength are estimated following the recommendation contained in [5], [6]. These documents provide two different methods:

- the Prediction method
- the Coverage method

In this dissertation, only the Coverage method is reported, because it allows to adopt any confidence value. The coverage method leads to the same results of the Prediction one adopting  $\alpha$  equal to 0.75.

In the Coverage method, the estimator  $\bar{x}_p$  of the lower fractile  $x_p$  is determined in such a way that

$$P(\bar{x}_p < x_p) = \alpha \quad (4-2)$$

Thus, the estimator  $\bar{x}_p$  is lower than the unknown fractile  $x_p$  with the probability (confidence)  $\alpha$ . In the following summary formulas are given without being derived.

If the standard deviation  $\sigma$  of the population is known, the estimator  $\bar{x}_p$  of the fractile  $p$  is given by

$$\bar{x}_p = m - \kappa_p \sigma \quad (4-3)$$

If the standard deviation  $\sigma$  of the population is unknown, the sample standard deviation  $s$  is considered

$$\bar{x}_p = m - k_p s \quad (4-4)$$

where  $m$  is the samples mean,  $\kappa_p$  and  $k_p$  are the estimation coefficients and they depend on the fractile probability  $p$ , the confidence level  $\alpha$  and the sample size  $n$ .

If the  $\sigma$  is known, then the estimation coefficient  $\kappa_p$  can be evaluated as

$$\kappa_p(p, \alpha, n) = -u_p + u_\alpha / \sqrt{n} \quad (4-5)$$

where  $u_p$  is  $p$ -fractile of standardised lognormal distribution,  $u_\alpha$  is  $\alpha$ -fractile of standardised lognormal distribution

If the  $\sigma$  is known, then the estimation coefficient  $\kappa_p$  can be evaluated as

$$k_p(p, \alpha, n) = -t(p, \alpha, n) / \sqrt{n} \quad (4-6)$$

where  $t(p, \alpha, n)$  is  $\alpha$ -fractile of the generalised noncentral t-distribution corresponding to the probability  $p$  and with  $v = n - 1$  DOFS.

These previous coefficients can incorporate the possible skewness  $\omega$  of a distribution. However, the r.v. in this thesis have not asymmetry  $\omega$ .

#### MEASUREMENT ERROR

Furthermore, existing structures differ from new ones as the data used in their assessment can be related to experimental measurements. Every time a measurement is done, its value is affected by error due to the accuracy of the instrument, the environmental conditions, and the operator execution. Consequently, the measurement error introduces an additional variability of the samples, leading to a higher dispersion in the parameter  $\bar{\vartheta}_d$  and to a further dispersion of possible values caught from the probability distribution of the basic variables. The outcomes of the tests carried out to evaluate the sample  $x_i$  is a r.v.  $X_{err}$  that can be expressed as

$$X_{err} = x_i \cdot \varepsilon(c_{v_\varepsilon}) \quad (4-7)$$

where  $\varepsilon(c_{v_\varepsilon})$  is a log-normally distributed random variable, having zero as log-mean and coefficient of variation equal to  $c_{v_\varepsilon}$ .

### 4.3 EFFECT OF KNOWLEDGE UNCERTAINTIES ON THE ESTIMATION OF DESIGN PARAMETERS

In this section, practical examples show the effect of knowledge uncertainties on the estimation of the design parameters. Each example considers the condition of known or unknown  $\sigma$  of the population. The effect of measurement error is treated separately.

#### ESTIMATION OF THE MEAN VALUE

An experimental campaign is simulated to estimate the mean value of the wires cross-sectional area  $A_p$ , utilizing samples caught from its log-normal distribution. This distribution is characterized by a mean value  $A_{pm}$  of  $12.30 \text{ cm}^2$  and  $c_p$  of 0.10. The MC simulation method is employed (Section 2.5.3.1). The results are reported in Figure 4-2 and show the *pdf* of the estimated mean value of the wires area  $\bar{A}_p$  for both  $\sigma$  population known or unknown conditions. The results are discussed first with reference to the different number of tests  $n$ , and successively by analysing the effect of variation of confidence  $\alpha$  within the considered range.

When the  $\sigma$  is known, increasing  $n$  reduces the dispersion of the probability density functions (*pdfs*), as shown in Eq. (A-46), and the mean value get closer to the reference value of  $A_p$ . This behaviour is expected, as a higher number of tests leads to a better evaluation of model uncertainty. On the other hand, reducing  $n$  produces the opposite effects. Additionally, the choice of confidence level  $\alpha$  also affects the estimation. By fixing  $n$  and increasing  $\alpha$ , the mean value of the *pdfs* shifts to the left, providing an average value that is always smaller than the reference one. This reduction is more pronounced with a small  $n$ .

When the  $\sigma$  is unknown, similar considerations apply, but with some differences compared to the previous. By fixing  $n$  and increasing  $\alpha$ , the mean value of the *pdfs* decreases more significantly than in the case of known  $\sigma$ . For the same condition of  $n$  and  $\alpha$  the  $t$  values are higher, leading to more conservative estimates. Moreover, the standard deviation of *pdfs* is not constant, because using the standard deviation of sample to estimate the parameter introduces a higher dispersion in the probability distribution. This behaviour is more pronounced with small  $n$ . As a matter of fact,  $\sigma$  unknown is the most frequent condition in the case of existing bridges where no information is often available about the mechanical properties.

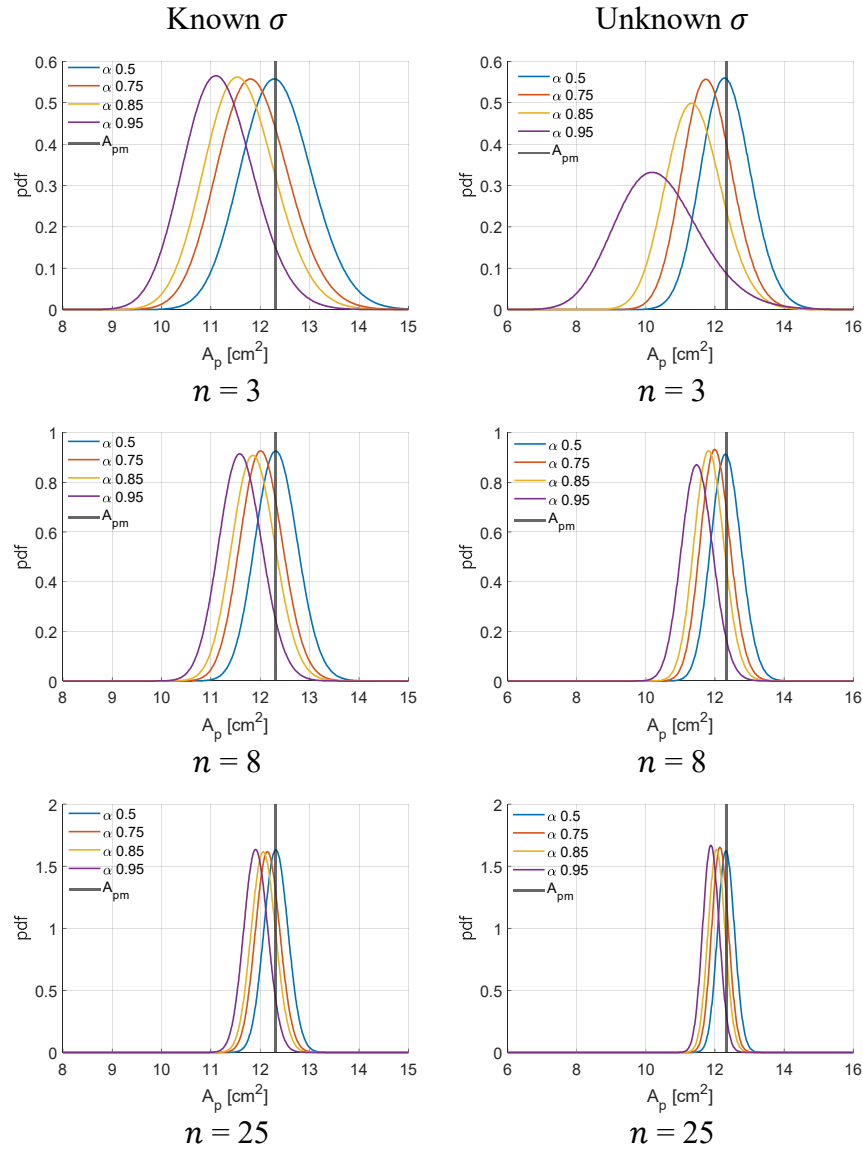


Figure 4-2 pdfs of the estimated  $\bar{A}_{pm}$ , for different values of  $\alpha$  and  $n$  in the case of known or unknown  $\sigma$  of population

In addition to previous uncertainties, the following results (Figure 4-3) consider the impact of measurement error ( $c_{v_\epsilon} = 0.10$ ) on the estimation only in the case  $\sigma$  unknown since it is the worst possible scenario. It is evident that the measurement error always leads to a safer estimation of  $\bar{A}_p$  than those showed in Figure 4-3. Moreover, the standard deviations of pdfs are greater than the previous. In particular, for low value of  $n$  and high  $\alpha = 0.95$  the  $\bar{A}_p$  pdf can reach value two times lower than the reference value, aspect that can notably influence the safety assessment.

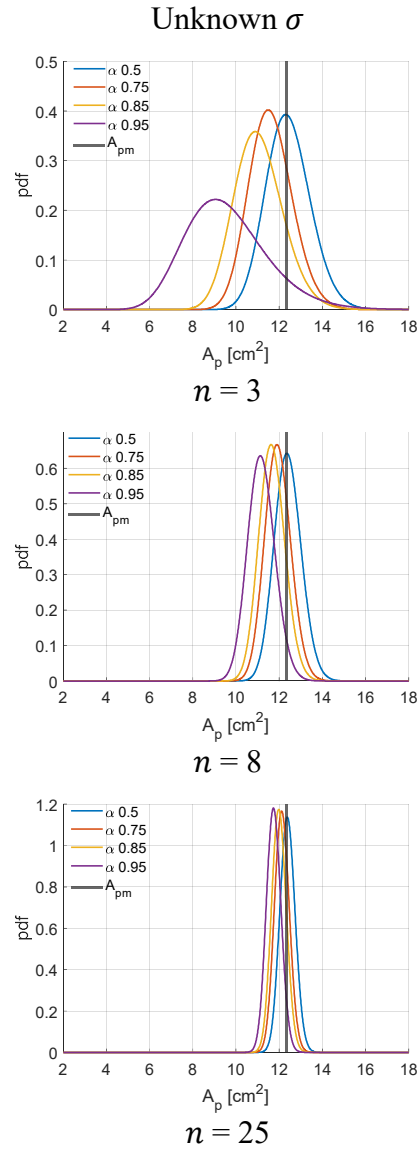


Figure 4-3 pdfs of the estimated  $\bar{f}_{ck}$ , for different values of  $\alpha$  and  $n$  considering measurement error

#### ESTIMATION OF THE CHARACTERISTIC VALUE

An experimental campaign is simulated to estimate the characteristic value of the concrete strength  $f_{ck}$ , utilizing samples caught from its log-normal distribution. This distribution is characterized by a mean value  $f_{cm}$  of 38 MPa and  $c_v$  of 0.10. The MC simulation method is employed. The results are reported in Figure 4-4 and show the pdfs of the estimated mean value of the wires area  $\bar{f}_{ck}$  for both the case of  $\sigma$  population known and unknown. The results are discussed first with reference to the different number of tests  $n$ , and successively by analysing the effect of varying the confidence  $\alpha$  within the considered range.

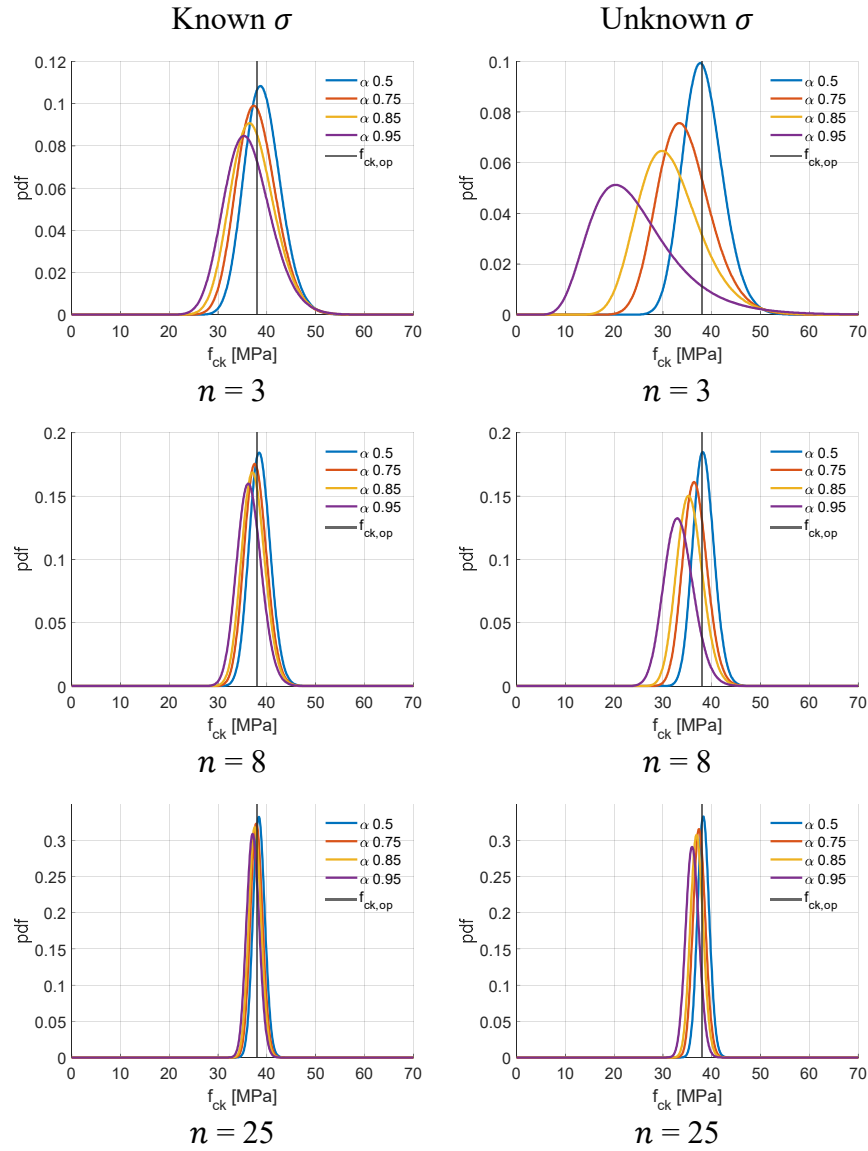


Figure 4-4 pdfs of the estimated  $\bar{f}_{ck}$ , for different values of  $\alpha$  and  $n$  in the case of known or unknown  $\sigma$  of population

Previous considerations drawn when adopting different  $n$  and  $\alpha$  are also valid for the results shown in Figure 4-4. It is important to highlight that the Coverage method for unknown  $\sigma$  provides, for low  $n$  and high  $\alpha$ , values from the safety side. For  $n = 3$  and  $\alpha = 0.95$ , the most probable value is two times lesser than the reference case. Moreover, the *pdf* is even characterized by values fourth time lesser. By increasing the number of tests, the coefficient  $k_p$ , Eq. (4-7), reduces and higher estimates are obtained.

This behaviour is further highlighted when considering the measurement error in Figure 4-5. In this scenario, the mean value of the distribution can be 75% lower than the reference one for high confidence levels ( $\alpha = 0.95$ ), with some values that tend to zero. By considering a broader perspective it is interesting to understand

how these values impact on the safety checks. The estimated safety ratio  $\bar{y}$  will be very low respect the reference one.

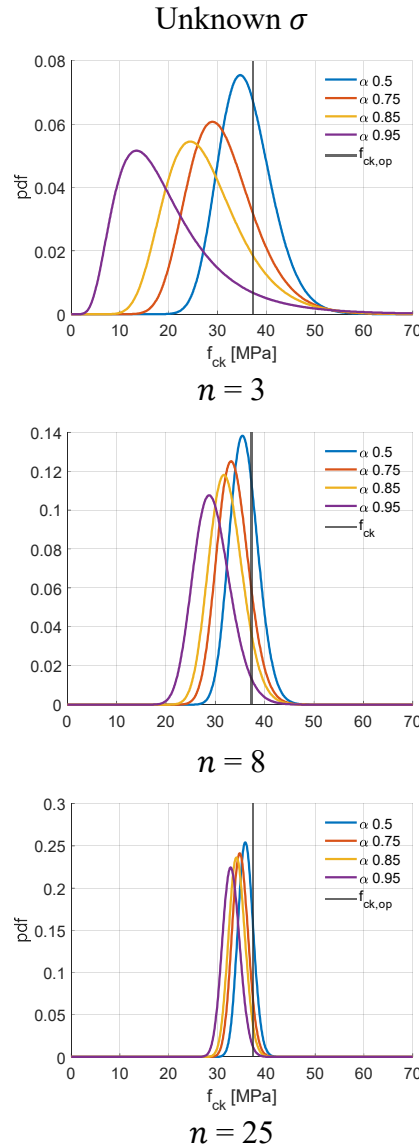


Figure 4-5 pdfs of the estimated  $\bar{f}_{ck}$ , for different values of  $\alpha$  and  $n$  considering measurement error

#### 4.4 FULLY PROBABILITISTIC EVALUATION

In addition to the semi-probabilistic approach, reliability assessment can be also performed adopting a fully probabilistic approach. In this approach, it is necessary to define the statistical properties of the basic random variables  $\mathbf{X}$ , represented by their probability density function  $f_X(\mathbf{x})$ , in order to establish the limit state function  $Z(\mathbf{X})$  and to calculate the probability of failure  $P_f$ .

For new structures, the statistical properties of the basic variables can often be derived from established standards or literature studies, as discussed in Section 3.4.

For existing structures, such as those analysed in this study, the distribution  $f_X(\mathbf{x})$  can only be estimated from a limited number of tests, which are often imprecise. As a result, due to the presence of knowledge uncertainties, the limit state function itself may vary, leading to an estimated probability of failure, denoted as  $\bar{P}_f$ .

In this analysis, the influence of knowledge uncertainty is considered only in relation to the strength of the post-tensioned beam. The statistical properties of the strength-related variables, based on the available test samples, are calculated following the recommendations provided in the Appendix A, which includes the estimation of both the mean value and the standard deviation.

## 4.5 REFERENCES

- [1]Fib, 2016. Bulletin N°80. Partial Safety factor Methods for Existing Structures, Lausanne.
- [2]Ditlevsen, O., Madsen, H.O., 1996. Structural reliability methods. Wiley, Chichester ; New York.
- [3]CEN, 2002. EN 1990 Eurocode 0- Basis of Structural Design., Brussels
- [4]CEN, 2005. EN 1991. Eurocode 1 – Actions on Structures, Brussels
- [5]Leonardo da Vinci Pilot Project CZ/02/B/F/PP-134007, 2005 Handbook 2. Reliability Backgrounds, Guide to the basis of structural reliability and risk engineering related to Eurocodes, supplemented by practical example.
- [6]Holický M, Vorlíček M., 1996. Statistical Procedures for Design Assisted by Testing; <https://doi.org/10.5169/SEALS-56082>

## 5 CASE STUDY

### 5.1 INTRODUCTION

Chapter 3 introduces the post-tensioned elements, detailing the safety checks and the basic random variables necessary for the assessment. Chapter 4 focuses on the evaluation of existing structures, introducing both the knowledge uncertainties and the key tools needed to estimate the required parameter for the assessment. By knowing all these information, this chapter aims to inspect the impact of knowledge uncertainties on the assessment process and consequently on the safety assessment taking into account a real case study evaluated through three different knowledge scenarios. In the first scenario an ideal condition of “perfect knowledge” is assumed. Safety checks performed in this condition lead to the reference solution of safety ratios, that is assumed as the real one. In the second scenario the knowledge process is considered: the influence of the number of tests on the safety checks is studied, considering also different choices for the confidence level assumed for the estimation of the parameters. In the third scenario, the error affecting the experimental measures is introduced.

These scenarios are evaluated using both the semi-probabilistic and fully probabilistic approach. In the semi-probabilistic approach, the safety checks with reference to the failure conditions of bending moment and shear are both influenced by the precompression system and they are performed and compared through the resulting capacity/demand ratio. The probabilistic distribution of the capacity/demand ratio related to the knowledge process is obtained using a Monte Carlo simulation (Section 2.5.3.1). Different choices regarding number of tests, confidence levels and measure uncertainties are considered, and the resulting probabilistic distributions are discussed. Finally, the impact of knowledge uncertainties is extended in the case of probabilistic approach by estimating the probability of failure using the Subset simulation method (Section 2.5.3.2). The probability of failure considered here represents the annual likelihood that the effect of loads exceeds the system’s strength.

### 5.2 DESCRIPTION OF THE CASE STUDY

The case study is a typical post-tensioned bridge (Figure 5-1), with a simple support static scheme and span length equal to 30 m. It is located in Italy and built in 1970 following the next recommendations: Circ. N. 1398 21/01/1965 for prestressed beam and Circ. N. 384 14/02/1962 for bridge loads. The deck is made of 5 cast-in-place prestressed I-beams, a slab of 0.23 m thick, and 5 prestressed crossbeams. The original drawing of the cross section of the bridge is reported in

Figure 5-2. Every beam (Figure 5-3) is prestressed by 6 cables and each cable is composed of 34 strands with diameters of 7 mm. The prestressing procedure has been carried out in two different construction stages. In the first stage, the 4 lower cables have been tensioned after the curing of beam concrete. Later, after the slab and crossbeam casting, the 2 remaining cables, the upper ones, have been tensioned. The original drawing of the position of the cables along the beam (Figure 5-4) is reported below. It is important to underline that information about bridge geometry, material characteristics, construction stages, and prestressing force have been derived from both original drawings and design reports. The compressive cubic strength of the concrete used for the beam is  $R_{ck} = 45$  MPa, while the one used for the slab is equal to 40 MPa. FeB44k steel has been used for the rebars. Prestressing strands present a characteristic ultimate strength equal to  $f_{ptk} = 1800$  MPa. Each cable has a cross-section area equal to  $A_p = 12.32$  cm<sup>2</sup>. The mean value of the prestressing stress applied to the cables is equal to  $\sigma_{pt} = 1189.6$  MPa.

The actions  $E_d$  are evaluated according to [1] (Load Model 1). The SAP2000 [2] software has been used to determine the most relevant external effects on the deck. For the safety checks only the most loaded beam is examined.

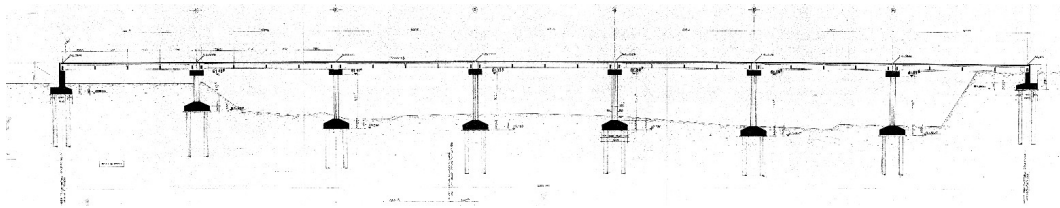


*Figure 5-1 Original longitudinal profile of the bridge*

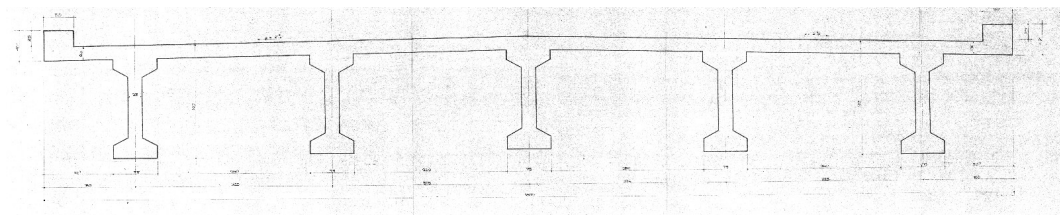


*Figure 5-2 Original cross section of the bridge deck*

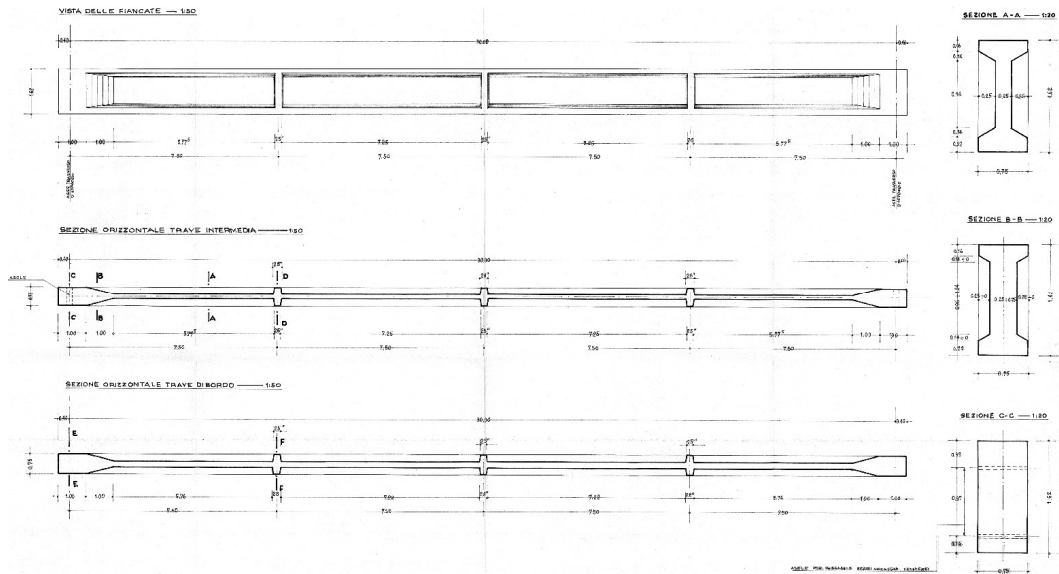


Figure 5-3 Side, horizontal and cross section original view of the beam

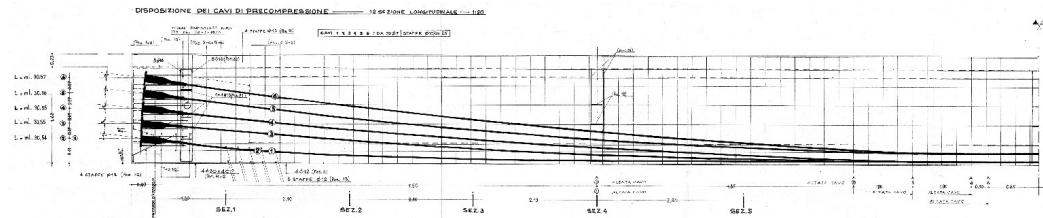


Figure 5-4 Original location of the strands along the beam

### 5.3 SEMI-PROBABILISTIC ANALYSIS AND SAFETY COEFFICIENT

In this section, either the adopted methods for the estimation of the set of the parameters  $\bar{\boldsymbol{\vartheta}}_d$  from the information received by experimental campaigns, and the way to consider the knowledge uncertainties are showed in an application concerning a real existing PT bridge by considering different combinations of  $n$ ,  $\alpha$  and  $c_{v_e}$ . First, the case study is introduced, then the three considered knowledge scenarios are described.

#### 5.3.1 PERFECT KNOWLEDGE SCENARIO

The perfect knowledge scenario is a theoretical condition that could be hypothetically achieved if an infinite number of tests are carried out, and if test procedures are not affected by measurement errors. In such an ideal scenario, the probability distributions of the basic variables are perfectly known and consequently the components of the vector  $\boldsymbol{\vartheta}_d$  are deterministic. The characteristic values of the material strengths are the 5% percentile of their probability distributions, according to the current safety format [3]. For what concerns the cables area, the nominal value has been used as suggested by the Standard [3]. For

the prestressing action, the Standard [3] recommends using either the mean value  $P_m$  or, if relevant, the upper  $P_{k,up}$  and lower  $P_{k,low}$  characteristic values. In this work, the mean value is adopted. Starting from the initial prestressing force and subtracting the immediate and long-term prestress losses, the nominal value of the prestressing force, is obtained for each cable. The resultant cable force has been computed all along the entire beam.

The shear check  $y_{d,s}$ , in the first cracked section that usually is the worst section, and bending check  $y_{d,b}$  are conducted following Section 3.3.2. The envelope check  $y_d$ , which is the minimum value between  $y_{d,s}$  and  $y_{d,b}$ , is also considered. In this scenario,  $y_{d,s}$ ,  $y_{d,b}$ ,  $y_d$  are deterministic results and they are used for the comparison with the following scenarios simulating the experimental campaigns.

Table 5-1 shows the probability distributions of  $\mathbf{X}$  and their related parameters  $\boldsymbol{\vartheta}_d$  in the perfect knowledge scenario. The mean values of the geometric basic variable ( $A_p$ ) and of the strengths of materials ( $f_c$ ,  $f_{up}$ ,  $f_{ys}$ ) are obtained from the information specified in the original technical drawings (terms with the subscript ‘op’ in Table 5-1), while their theoretical coefficients of variation are acquired from the literature. The mean value of the current prestressing force,  $P_{pt}$ , is obtained by subtracting from the initial prestressing force,  $P_{pt,i}$ , declared in the original design, the prestress losses  $\Delta P_{pt}$  (either long-term and immediate ones), calculated following the approach proposed in [4]. For  $P_{pt}$  a unique coefficient of variation  $c_{v_p}$ , is adopted, it includes the uncertainties related to the initial force applied to the cables, as well as uncertainties related to immediate and long-term losses. The global coefficient of variation  $c_{v_p}$  can be obtained according to [5], as follows:

$$c_{v_p} = \frac{\sqrt{(c_{v_F} \cdot \mu_F)^2 + (c_{v_{losses\_lt}} \cdot \mu_{losses\_lt})^2 + (c_{v_{losses\_st}} \cdot \mu_{losses\_st})^2}}{\mu_F + \mu_{losses\_lt} + \mu_{losses\_st}} \quad (5-1)$$

where  $c_{v_F}$  and  $\mu_F$  are the coefficient of variation and the mean value of initial prestressing force, respectively;  $c_{v_{losses\_lt}}$  and  $\mu_{losses\_lt}$  are , the coefficient of variation and the mean value of long-term prestress losses, respectively;  $c_{v_{losses\_st}}$  and  $\mu_{losses\_st}$  are the immediate ones. All the coefficients of variation of Eqn. (5-1) have been obtained with reference to Table 3-10 and they have been assumed constants and equal to their mean values. For the application of the safety format, the equivalent cable of the post-tensioned system is adopted.

Once the parameters  $\boldsymbol{\vartheta}_d$  are obtained, the safety ratios  $y_d$  for the perfect knowledge scenario can be determined and they are equal to:

- $y_{d,s} = V_{rd}/V_{ed} = 0.97$

- $y_{d,b} = M_{rd}/M_{ed} = 1.09$
- $y_d = \min(y_{d,s}, y_{d,b}) = 0.97$

As can be seen, an existing bridge case study representative of a common example in Italy designed according to outdated codes actually result in a bridge with performance close to a new one. The reason is that in the past bridges are designed with very high vertical loads (military loads).

*Table 5-1 Probability distributions of the basic variables and related design parameters.*

| Basic variables          | Distribution type | $\mu$                                                    | $c_v$                   | Design parameters                                          |
|--------------------------|-------------------|----------------------------------------------------------|-------------------------|------------------------------------------------------------|
| $f_c$                    | Log-Normal        | $f_{cm} = R_{ck,op} \cdot 0.83 + 8$<br>[MPa]<br>[3]      | 0.10<br>Table 3-2       | $f_{cd} = \frac{f_{ck}}{\gamma_c} = 21.7$<br>[MPa]         |
| $f_{up}$                 | Log-Normal        | $f_{upm} = f_{uk,op} \cdot 1.04$<br>[MPa]<br>Table 3-7   | 0.025<br>Table 3-7      | $f_{upd} = \frac{f_{upk}}{\gamma_s}$<br>$= 1475$<br>[MPa]  |
| $f_{ys}$                 | Log-Normal        | $f_{ysm} = f_{yk,op} + 2 \cdot 30$<br>[MPa]<br>Table 3-4 | 0.025<br>Table 3-4      | $f_{ysd} = \frac{f_{ysk}}{\gamma_s}$<br>$= 305.6$<br>[MPa] |
| $A_p$<br>(single strand) | Log-Normal        | $A_{pm} = A_{pm,op}$<br>[3]                              | 0.02<br>Sec.<br>3.4.1.3 | $A_{pm} = 12.32$<br>[cm <sup>2</sup> ]                     |
| $P_{pt}$                 | Log-Normal        | $P_{ptm} = P_{pti,op} - \Delta P_{pt}$                   | 0.08<br>Eqn.<br>(5-1)   | $P_{ptm} = 1221.81$<br>[kN]<br>(mid-span)                  |

### 5.3.2 PARTIAL KNOWLEDGE SCENARIO WITHOUT MEASUREMENT ERRORS

The partial knowledge scenario refers to a condition in which only a reduced number of tests is carried out, and a variability of  $\bar{\theta}_d$  arises due to the uncertain estimate of basic variables distribution. Such a scenario is simulated starting from the theoretical probability distribution  $f_X(\mathbf{x})$  of the random variables  $\mathbf{X}$ , and from the fixed number of tests  $n$  chosen. Many test campaigns involving the same number of tests are simulated; each simulation, composed by a different set of samples, leads to a different estimation of the probability distribution parameters, given the confidence factor  $\alpha$ , and consequently to different values  $\bar{\theta}_d$  of the estimated r.v.  $\bar{\theta}_d$ .

The parameters are estimated through the recommendations contained in [3] following the conditions of unknown population variance  $\sigma$  of the basic variable distributions. The characteristics values of materials ( $f_{ck}$ ,  $f_{ysk}$ ,  $f_{upk}$ ) are estimated through the Coverage method Section 4.2. The mean values of the wires area ( $A_{pm}$ ) and of the prestressing force  $P_{pt\_m}$ , derived from samples, are estimated using the t-Student method Appendix A.

Once the parameters  $\bar{\theta}_d$  for each simulation have been calculated, it is possible to carry out the safety checks and to obtain the safety measure  $\bar{y}_d$ . A Monte Carlo simulation (Section 2.5.3.1) is applied to obtain the probability distribution of the safety ratio  $\bar{Y}_d$ . With the aim at evaluating how much uncertainties on the design parameters affect the estimated  $\bar{y}_d$ , the probability to stay into the safe domain  $\bar{y}_d < y_d$  is calculated.

### 5.3.3 PARTIAL KNOWLEDGE SCENARIO WITH MEASUREMENT ERRORS

In this scenario, the  $\bar{\theta}_d$  are obtained in a similar way, but in this case the measurement error is also considered. The error is added to each sample value, by considering a probability distribution of the erroneous measurement having a coefficient of variation  $c_{v,\varepsilon}$  and a mean value equal to the one of the samples itself (Figure 5-5).

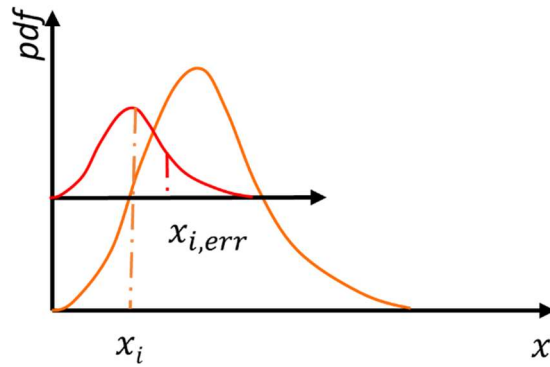


Figure 5-5 Measurement error

## 5.4 PARAMETRIC ANALYSIS AND RELATED RESULTS

This section presents the evaluation of the safety ratio  $y_d$  and its variability in the two scenarios simulating the execution of experimental campaigns.

In both scenarios, multiple analyses have been carried out by choosing different number  $n$  of tests and by exploring the full  $\alpha$  range [0.50-0.95] with the aim of observing how  $F_{\bar{Y}_d}(y_d)$  changes. It is worth to recall that  $\alpha = 0.75$  is the value

suggested by codes [6]. The following hypotheses have been assumed in the parametric analysis: in the second knowledge scenario, the same values of  $n$  and  $\alpha$  are assumed for all the parameters  $\bar{\boldsymbol{\theta}}_d$  and no measurement error is considered. In the third knowledge scenario, the measurement error is added, by considering different values of the coefficient of variation  $c_{v_\epsilon}$ .

#### 5.4.1 OUTCOMES OF THE ASSESSMENT THROUGH TESTS WITHOUT MEASUREMENT ERROR

Such a scenario simulates a possible experimental campaign conducted on the case study without considering the measurement error. The results obtained are reported in Figure 5-6 and in Figure 5-7. Figure 5-6 shows the *pdfs* of the shear safety ratio  $\bar{y}_{d,s}$ , of the bending one  $\bar{y}_{d,b}$ , and the *pdfs* of their envelopes  $\bar{y}_d$ , obtained for different values of  $n$  and  $\alpha$ . To make the comparison easier, for all the three safety ratios the nondimensional ratio  $\bar{y}_d/y_d$  is reported. In this way, the value 1 is the boundary between estimations of the capacity/demand ratio higher or lower than the reference case of perfect knowledge.

Figure 5-7 reports, instead, the trends of the *CDF* of  $\bar{Y}_d$  evaluated at the reference value,  $F_{\bar{Y}_d}(y_d)$ , and of the mean value  $\bar{y}_{dm} = E[\bar{Y}_d]$  in the nondimensional form, by varying the confidence level  $\alpha$  and the number of tests  $n$ . It is important to underline that the choice of the partial factors do not affect the nondimensional reported results, since they appear both on  $\bar{y}_d$  and  $y_d$ .

With the aim of understanding the results of this scenario in a clearer way, Table 5-2, Table 5-4 and Table 5-6 display the numerical values obtained for the probability of estimating values of the safety ratio lower than the reference one  $F_{\bar{Y}_d}(y_d)$ , for the case of shear, bending, and envelope checks, while Table 5-3, and Table 5-7 contain the numerical results relevant to the ratio  $\bar{y}_d/y_d$ .

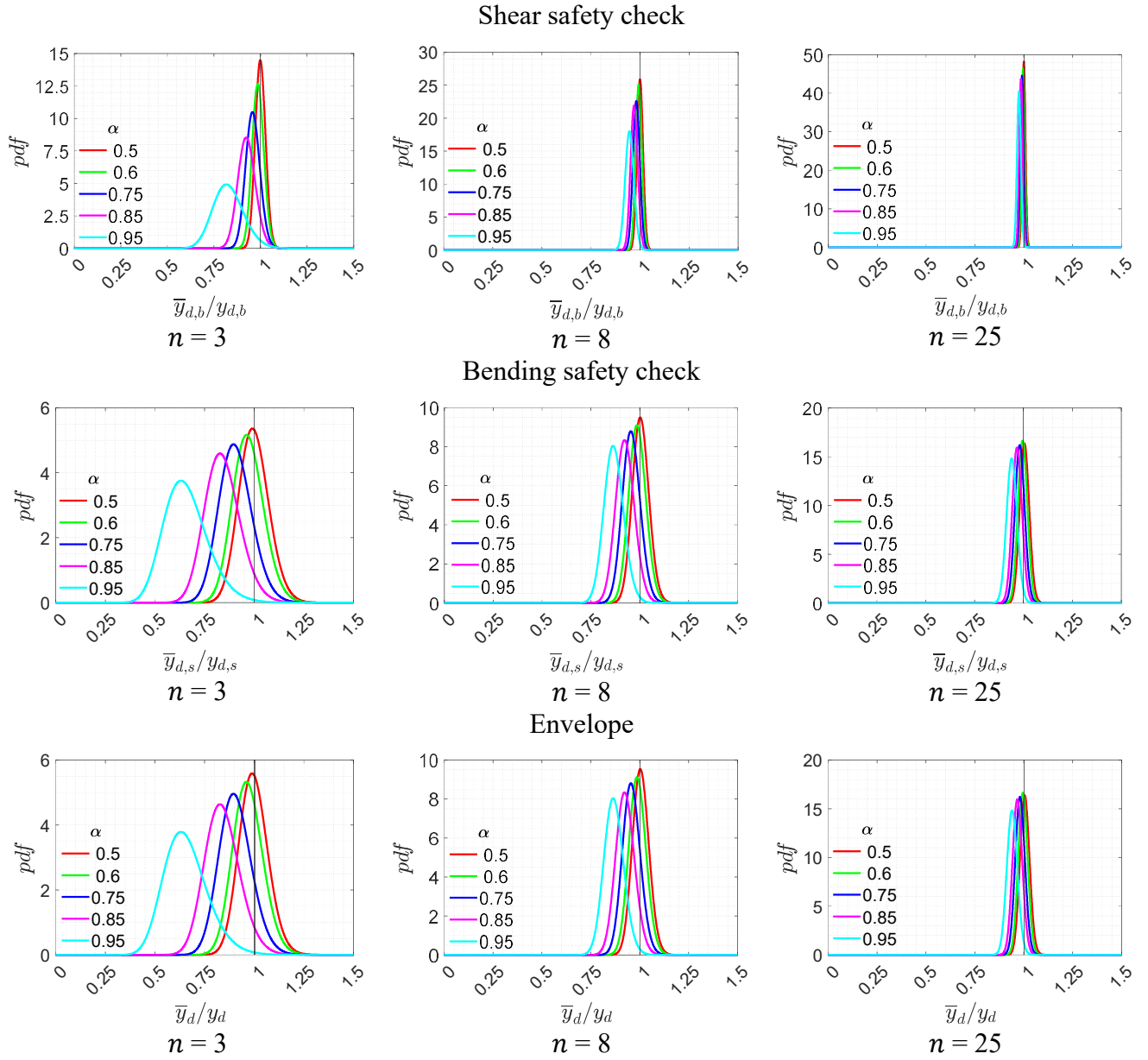


Figure 5-6 pdfs of the ratio  $\bar{y}_d/y_d$ , for different values of  $\alpha$  and  $n$ .

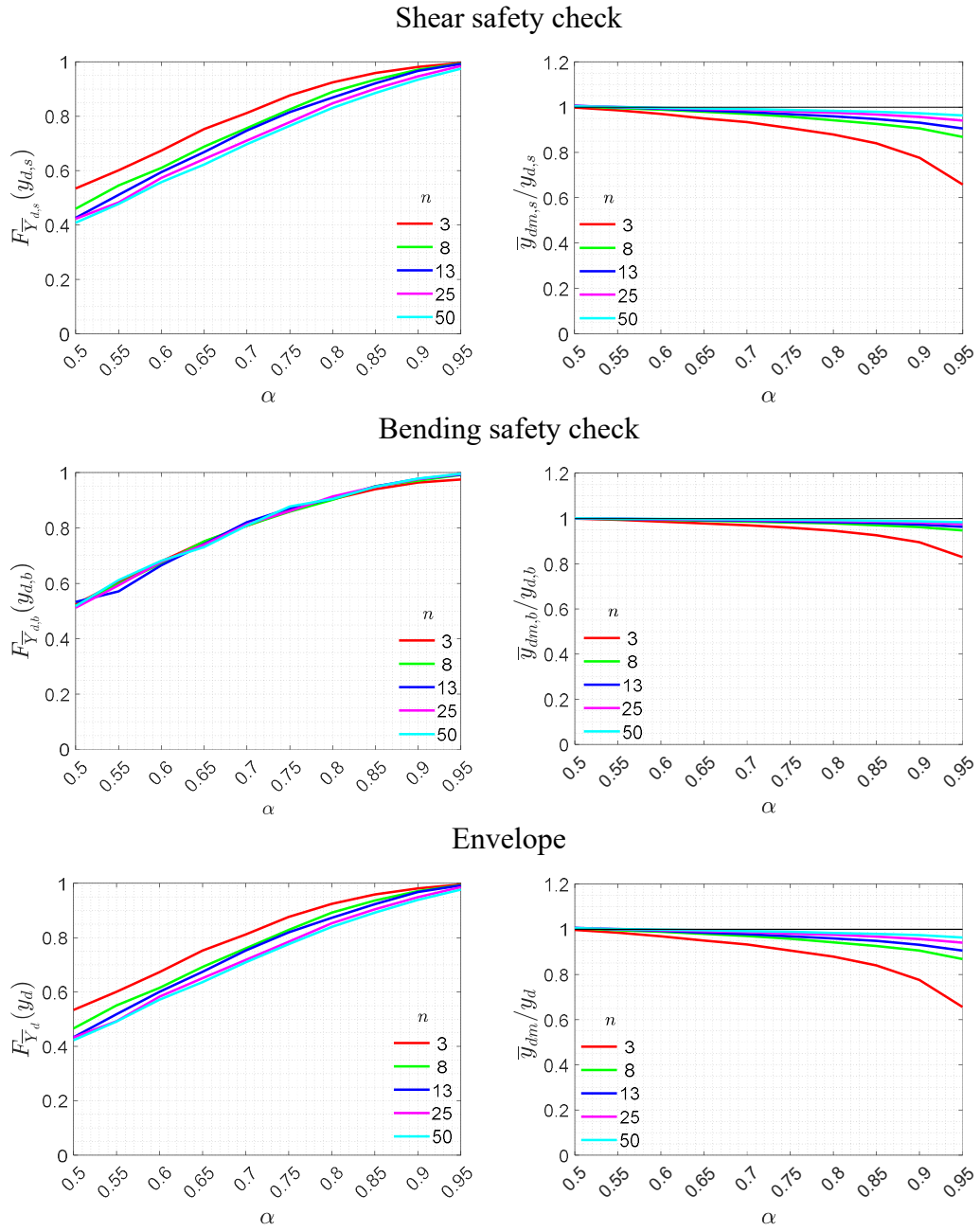


Figure 5-7 Diagrams of  $F_{\bar{Y}_d}(y_d)$  and  $\bar{y}_{dm}/y_d$  with  $\alpha$  and  $n$  for shear, bending and envelope checks.

Table 5-2 Numerical values of  $F_{\bar{Y}_{d,s}}(y_{d,s})$  for different values of  $\alpha$  and  $n$

| $F_{\bar{Y}_{d,s}}(y_{d,s})$ | $\alpha$    |             |             |             |             |
|------------------------------|-------------|-------------|-------------|-------------|-------------|
|                              | <b>0.50</b> | <b>0.60</b> | <b>0.75</b> | <b>0.85</b> | <b>0.95</b> |
| <b>3</b>                     | 0.53        | 0.67        | 0.87        | 0.95        | 0.99        |
| <b>n 8</b>                   | 0.46        | 0.61        | 0.82        | 0.93        | 0.99        |
| <b>25</b>                    | 0.42        | 0.57        | 0.78        | 0.99        | 0.99        |

Table 5-3 Numerical values of  $\bar{y}_{dm,s}/y_{d,s}$  for different values of  $\alpha$  and  $n$ .

| $\bar{y}_{dm,s}/y_{d,s}$ | $\alpha$    |             |             |             |             |
|--------------------------|-------------|-------------|-------------|-------------|-------------|
|                          | <b>0.50</b> | <b>0.60</b> | <b>0.75</b> | <b>0.85</b> | <b>0.95</b> |
| <b>3</b>                 | 1           | 0.97        | 0.90        | 0.84        | 0.66        |
| <b>n 8</b>               | 1           | 0.99        | 0.96        | 0.92        | 0.87        |
| <b>25</b>                | 1           | 0.99        | 0.98        | 0.97        | 0.94        |

Table 5-4 Numerical values of  $F_{\bar{Y}_{d,b}}(y_{d,b})$  for different values of  $\alpha$  and  $n$ .

| $F_{\bar{Y}_{d,b}}(y_{d,b})$ | $\alpha$    |             |             |             |             |
|------------------------------|-------------|-------------|-------------|-------------|-------------|
|                              | <b>0.50</b> | <b>0.60</b> | <b>0.75</b> | <b>0.85</b> | <b>0.95</b> |
| <b>3</b>                     | 0.52        | 0.68        | 0.86        | 0.94        | 0.98        |
| <b>n 8</b>                   | 0.52        | 0.67        | 0.86        | 0.94        | 0.99        |
| <b>25</b>                    | 0.51        | 0.68        | 0.86        | 0.95        | 0.99        |

Table 5-5 Numerical values of  $\bar{y}_{dm,b}/y_{d,b}$  for different values of  $\alpha$  and  $n$ .

| $\bar{y}_{dm,b}/y_{d,b}$ |    | $\alpha$ |      |      |      |      |
|--------------------------|----|----------|------|------|------|------|
|                          |    | 0.50     | 0.60 | 0.75 | 0.85 | 0.95 |
| <b>n</b>                 | 3  | 1        | 0.98 | 0.96 | 0.93 | 0.83 |
|                          | 8  | 1        | 0.99 | 0.98 | 0.97 | 0.95 |
|                          | 25 | 1        | 0.99 | 0.99 | 0.99 | 0.97 |

Table 5-6 Numerical values of  $F_{\bar{y}_d}(y_d)$  for different values of  $\alpha$  and  $n$ .

| $F_{\bar{y}_d}(y_d)$ |    | $\alpha$ |      |      |      |      |
|----------------------|----|----------|------|------|------|------|
|                      |    | 0.50     | 0.60 | 0.75 | 0.85 | 0.95 |
| <b>n</b>             | 3  | 0.53     | 0.67 | 0.88 | 0.96 | 0.99 |
|                      | 8  | 0.47     | 0.62 | 0.83 | 0.94 | 0.99 |
|                      | 25 | 0.43     | 0.58 | 0.78 | 0.90 | 0.97 |

Table 5-7 . Numerical values of  $\bar{y}_{dm}/y_d$  for different values of  $\alpha$  and  $n$ .

| $\bar{y}_{dm}/y_d$ |    | $\alpha$ |      |      |      |      |
|--------------------|----|----------|------|------|------|------|
|                    |    | 0.50     | 0.60 | 0.75 | 0.85 | 0.95 |
| <b>n</b>           | 3  | 1        | 0.97 | 0.91 | 0.84 | 0.66 |
|                    | 8  | 1        | 0.99 | 0.96 | 0.93 | 0.87 |
|                    | 25 | 1        | 0.99 | 0.98 | 0.97 | 0.94 |

The results reported in the previous figures (Figure 5-6 and Figure 5-7) and tables (Table 5-2-Table 5-7) are discussed first with reference to the different number of tests  $n$ , and successively by analysing the effect of variations of confidence  $\alpha$  within the range considered.

Different choices on the number of tests  $n$  have a significant impact on the results obtained. By comparing the results of Figure 5-6 it is observed that by increasing the number of tests, the dispersion of the probability distributions reduces, and the mean value get closer to the reference scenario. This behaviour is expected as a higher number of tests leads to a better evaluation of the model uncertainties. On the other hand, by reducing  $n$ , the probability distributions move away from the reference scenario toward the safety zone (i.e. a capacity/demand ratio estimation lower than the reference scenario), but, at the same time, they are characterized by

a greater standard deviation. Moreover, the shear dispersion is generally greater than the bending one. This aspect can be explained by the fact that the number of r.v. involved in  $y_{d,b}$  (bending) is lower than the number of the r.v. involved in  $y_{d,s}$  (shear). The different choices of  $n$  result in changes in the probability distributions, which in turn, affect the probability  $F_{\bar{y}_d}(y_d)$  of staying within the safety zone  $\bar{y}_d < y_d$ .

For what concerns the results for the shear check depicted in Figure 5-7 (first row), taking a low value of  $n$  leads to a larger, i.e. safer, value of  $F_{\bar{y}_{d,s}}(y_{d,s})$  and to a conservative value of  $\bar{y}_{dm,s}/y_{d,s}$  because the methods used to calculate  $\bar{\theta}_d$  (see Eqn. (4-5) and Eqn. (A-54)) produce conservative values of the estimated parameters thanks to the coefficient  $k_p$ , (see Eqn. (4-6), and to the percentile  $t_\alpha$  of the t-Distribution. This behaviour is also observed for the bending check (Figure 5-7) where the variation of  $n$  results in different values of  $\bar{y}_{dm,b}/y_{d,b}$ . As already observed in Figure 5-6, the different number of random variables involved in the estimation of  $y_{d,b}$  has an effect also in the trends of the ratio  $\bar{y}_{dm,b}/y_{d,b}$  of Figure 5-7, which are closer to each other with respect to those of the ratio  $\bar{y}_{dm,s}/y_{d,s}$ .

As expected, the choice of the confidence level  $\alpha$  also produces different results. By increasing  $\alpha$ , the mean value of the safety ratio distributions (Figure 5-6) shifts to the left, providing an average value that is always smaller than the reference one, i.e. in the safety zone. The safety check is statistically more precautionary, but at the same time, less accurate, since the dispersion increases, and a significant underestimation of the safety coefficient is likely to be obtained. The mean values reported in Figure 5-7 start from 1 for  $\alpha$  equal to 0.5 and decrease in precautionary manner as  $\alpha$  increases. This reduction is more pronounced with a small number of tests. At the same time, as already observed from the perspective of different  $n$ , the dispersion of the distribution  $\bar{y}_d/y_d$  increases. This aspect negatively influences the probability  $F_{\bar{y}_d}(y_d)$  to obtain values of  $\bar{y}_d$  lower than the reference value  $y_d$ . Once fixed  $\alpha$ , the trend of  $F_{\bar{y}_d}(y_d)$  does not excessively change by varying the number  $n$  of tests. By assuming as a reference value the confidence level  $\alpha$  equal to 0.75, as recommended by [3] and [5], the probability  $F_{\bar{y}_d}(y_d)$  is around 0.8 for different  $n$  and  $\alpha$  (Figure 5-7). In other words, using the current safety format suggested by the code, the capacity is generally underestimated of 20%.

#### 5.4.2 OUTCOMES OF THE ASSESSMENT THROUGH TESTS WITH MEASUREMENT ERROR

In addition to previous uncertainties related to test numbers and confidence factors, the current scenario also considers the impact of the measurement error on the safety assessment. Since in the literature there are no information about the measurement errors regarding tests for the survey of the prestressing system, in this work a credible coefficient of variation  $c_{v_\epsilon}$  equal to 0.10 is assumed for the basic

random variables selected in the previous sections. However, with the aim of providing a more general overview of the potential influence of this parameter for measurement systems more or less accurate than the previous ones, the main results have been also evaluated for different values of  $c_{v_\varepsilon}$ . In detail, results concerning  $c_{v_\varepsilon} = 0.05$  and  $c_{v_\varepsilon} = 0.20$  are reported and discussed at the end of this section.

The obtained results for  $c_{v_\varepsilon}$  equal to 0.10 are reported in Figure 5-8 and Figure 5-9. Figure 5-8 shows the differences between the probability distribution with and without measurement error, depicted by solid and dashed lines, respectively, for the same values of  $n$  adopted in the previous section (Figure 5-6). The results are reported in term of shear, bending and envelope checks.

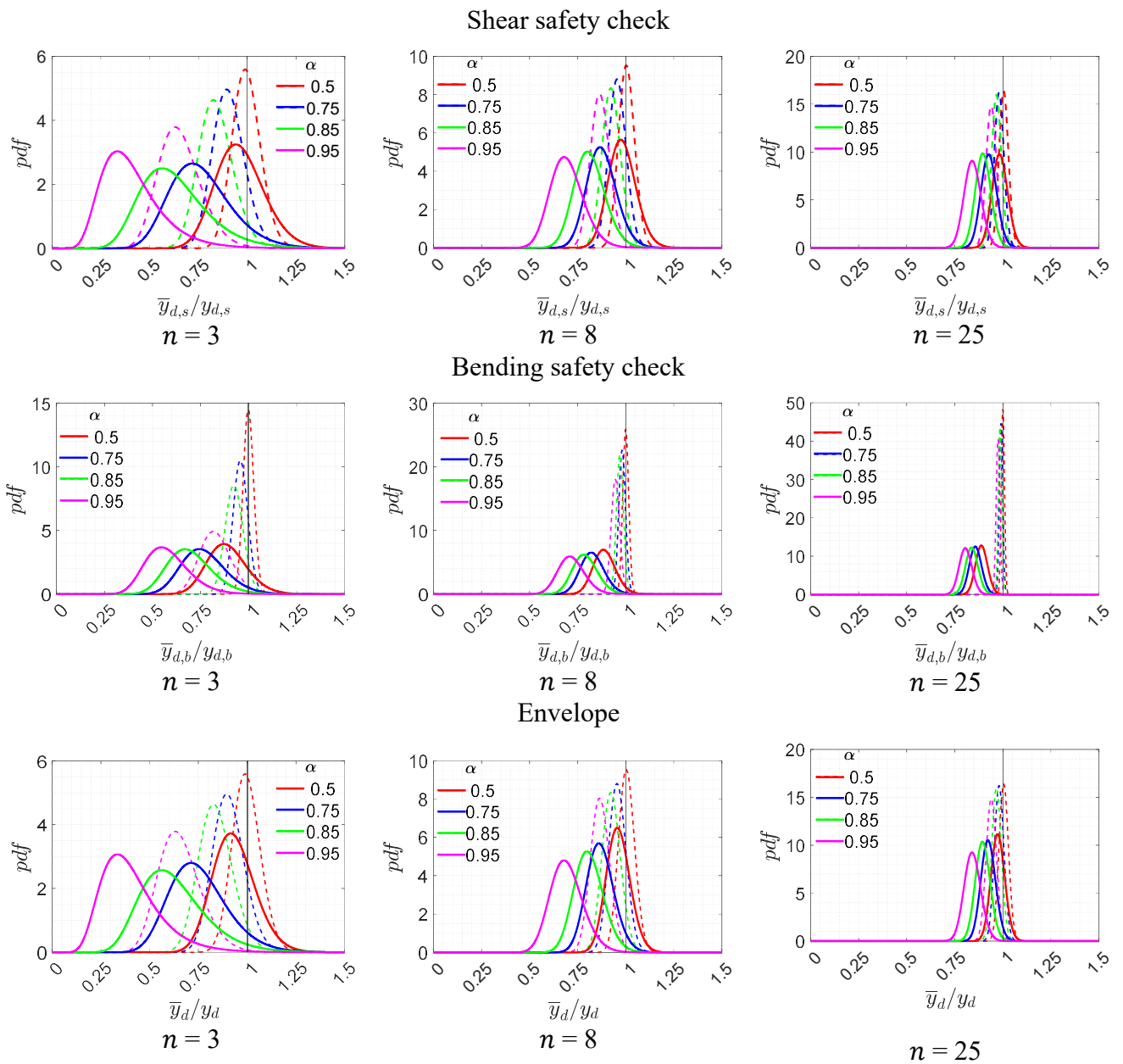


Figure 5-8 pdfs of the ratio  $\bar{y}_d/y_d$ , for different values of  $\alpha$  and  $n$  by considering the measurement error.

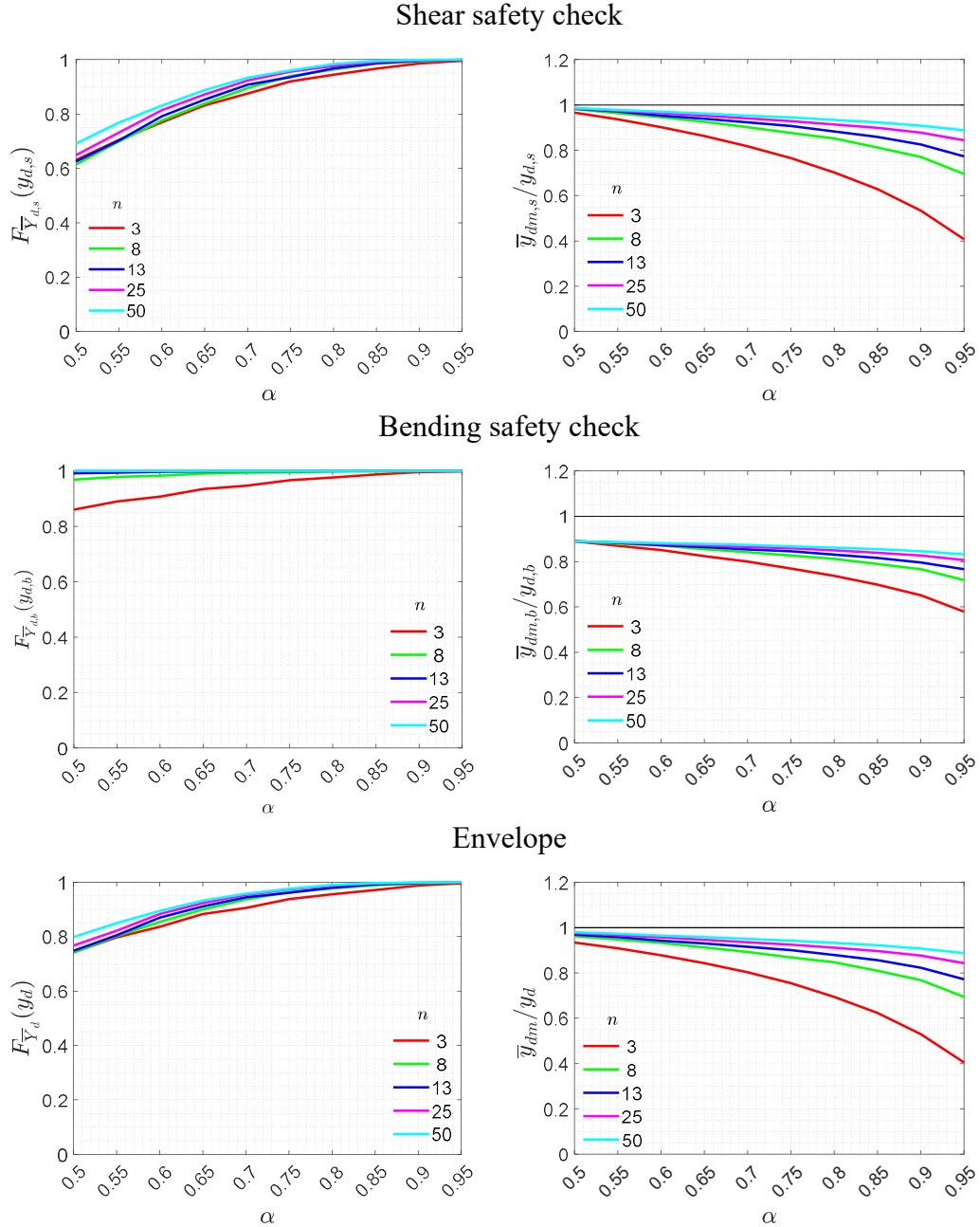


Figure 5-9 Diagram of  $F_{\bar{Y}_d}(y_d)$  and  $\bar{y}_{dm}/y_d$  with  $\alpha$  and  $n$  for shear, bending and envelope checks.

Figure 5-9 reports the trends of  $F_{\bar{Y}_d}(y_d)$  and the mean values  $\bar{y}_{dm}$ , obtained by varying  $n$  and  $\alpha$ . Even in this scenario, the probability of obtaining a bending check worse than the shear check is low.

With the aim of understanding the results of this scenario in a clearer way, Table 5-8, Table 5-10 and Table 5-12 display the numerical results for the shear, bending and envelope checks of  $F_{\bar{y}_d}(y_d)$ , while Table 5-9, Table 5-11, and contain those related to  $\bar{y}_d/y_d$ .

By analysing the results of Figure 5-8, it is evident that the measurement error always leads to a safer estimation of  $\bar{y}_d$  with respect to the results achieved in Figure 5-6 (no measurement error), although the same number of tests  $n$ , and with the same confidence levels  $\alpha$  are considered. Moreover, it can be observed that the values of the standard deviations are notably larger than those depicted in Figure 5-6 for all the failure conditions, and especially for the bending checks.

For what concerns the results of Figure 5-9, the measurement error provides higher values of probability  $F_{\bar{y}_d}(y_d)$  of obtaining values of  $\bar{y}_d$  lower than the reference ones. In the current scenario the lower value of the probability is always greater than 0.61 (shear check,  $n=8$ ,  $\alpha=0.5$ ), while without the measurement error the lower probability achieved is 0.42 (shear check,  $n=25$ ,  $\alpha=0.5$ ). Again, by assuming  $\alpha=0.75$  as a reference value for the confidence level, the probability  $F_{\bar{y}_d}(y_d)$  is at least equal or greater than 0.92 for different values of  $\alpha$  and  $n$ , while previously was around 0.80. Finally, the mean value  $\bar{y}_{dm}$  reduces to 0.40 (shear check,  $n=3$ ,  $\alpha=0.95$ ), with respect to the scenario without measurement error whose lower value is 0.66 (envelope check,  $n=3$ ,  $\alpha=0.95$ ). The observed trends are similar no matter what the values of  $\alpha$  and  $n$  are.

Table 5-8 Numerical values of  $F_{\bar{y}_{d,s}}(y_{d,s})$  for different values of  $\alpha$  and  $n$  with the measurement error.

| $F_{\bar{y}_{d,s}}(y_{d,s})$ | $\alpha$ |      |      |      |      |
|------------------------------|----------|------|------|------|------|
|                              | 0.50     | 0.60 | 0.75 | 0.85 | 0.95 |
| <b>3</b>                     | 0.63     | 0.77 | 0.92 | 0.97 | 0.99 |
| <b><i>n</i> 8</b>            | 0.61     | 0.78 | 0.94 | 0.98 | 0.99 |
| <b>25</b>                    | 0.65     | 0.81 | 0.95 | 0.99 | 0.99 |

Table 5-9 Numerical values of  $\bar{y}_{dm,s}/y_{d,s}$  for different values of  $\alpha$  and  $n$  with the measurement error.

| $\bar{y}_{dm,s}/y_{d,s}$ | $\alpha$ |      |      |      |      |
|--------------------------|----------|------|------|------|------|
|                          | 0.50     | 0.60 | 0.75 | 0.85 | 0.95 |
| <b>3</b>                 | 0.96     | 0.90 | 0.76 | 0.63 | 0.40 |
| <b><i>n</i> 8</b>        | 0.98     | 0.94 | 0.88 | 0.81 | 0.69 |
| <b>25</b>                | 0.98     | 0.96 | 0.93 | 0.89 | 0.84 |

Table 5-10 Numerical values of  $F_{\bar{y}_{d,b}}(y_{d,b})$  for different values of  $\alpha$  and  $n$  with the measurement error.

| $F_{\bar{y}_{d,b}}(y_{d,b})$ | $\alpha$ |      |      |      |      |
|------------------------------|----------|------|------|------|------|
|                              | 0.50     | 0.60 | 0.75 | 0.85 | 0.95 |
| <b>3</b>                     | 0.89     | 0.93 | 0.97 | 0.99 | 1    |
| <b><i>n</i> 8</b>            | 0.97     | 0.99 | 0.99 | 1    | 1    |
| <b>25</b>                    | 1        | 1    | 1    | 15   | 1    |

Table 5-11 Numerical values of  $\bar{y}_{dm,b}/y_{d,b}$  for different values of  $\alpha$  and  $n$  with the measurement error.

| $\bar{y}_{dm,b}/y_{d,b}$ | $\alpha$ |      |      |      |      |
|--------------------------|----------|------|------|------|------|
|                          | 0.50     | 0.60 | 0.75 | 0.85 | 0.95 |
| <b>3</b>                 | 0.89     | 0.85 | 0.77 | 0.70 | 0.58 |
| <b><i>n</i> 8</b>        | 0.89     | 0.87 | 0.83 | 0.79 | 0.72 |
| <b>25</b>                | 0.89     | 0.87 | 0.86 | 0.84 | 0.81 |

Table 5-12 Numerical values of  $F_{\bar{y}_d}(y_d)$  for different values of  $\alpha$  and  $n$  with the measurement error.

| $F_{\bar{y}_d}(y_d)$ | $\alpha$ |      |      |      |      |
|----------------------|----------|------|------|------|------|
|                      | 0.50     | 0.60 | 0.75 | 0.85 | 0.95 |
| <b>3</b>             | 0.75     | 0.83 | 0.93 | 0.97 | 0.99 |
| <b><i>n</i> 8</b>    | 0.74     | 0.85 | 0.96 | 0.99 | 0.99 |
| <b>25</b>            | 0.77     | 0.88 | 0.97 | 0.99 | 0.99 |

Table 5-13 Numerical values of  $\bar{y}_{dm}/y_d$  for different values of  $\alpha$  and  $n$  with the measurement error.

| $\bar{y}_{dm}/y_d$ | $\alpha$ |      |      |      |      |
|--------------------|----------|------|------|------|------|
|                    | 0.50     | 0.60 | 0.75 | 0.85 | 0.95 |
| <b>3</b>           | 0.93     | 0.88 | 0.75 | 0.62 | 0.41 |
| <b>n 8</b>         | 0.96     | 0.91 | 0.87 | 0.81 | 0.69 |
| <b>25</b>          | 0.98     | 0.96 | 0.92 | 0.89 | 0.84 |

Aiming at observing how much the structural safety is affected by different measurement errors, the results for  $c_{v_e}$  equal to 0.05 (Figure 5-10 a) and 0.20 (Figure 5-10 b) are reported only for  $\bar{y}_{dm}/y_d$  relevant to the envelope case. Figure 5-10 shows that the measurement error significantly influences the final value of the estimated capacity mean  $\bar{y}_{dm}$ .

It is also worth to observe that, by increasing the measurement error variability, the probability to have a safe estimation of  $\bar{y}_d$  increases. The uncertainty due to the measurement error introduces an extra variability of the measured quantity that implies a further safety on the final results. This is a remarkable difference between new structures and existing ones, and it should be considered within the safety format to avoid over-conservative safety assessments.

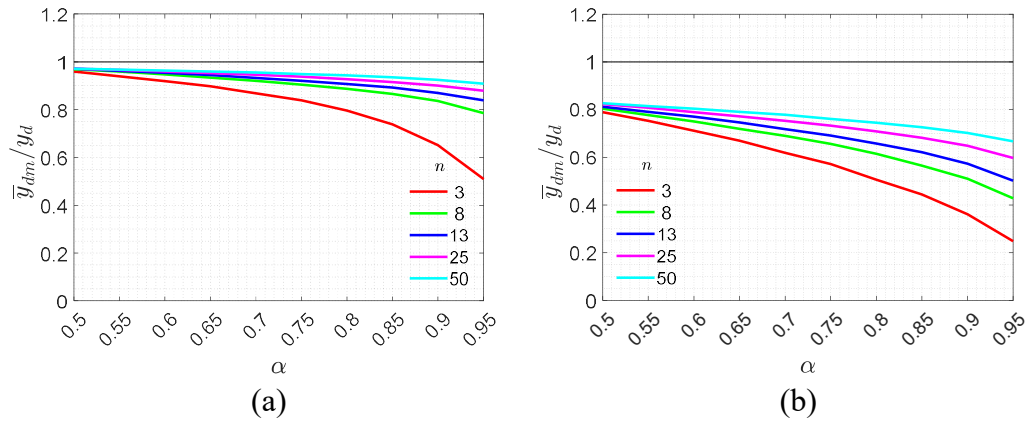


Figure 5-10 Diagram of  $\bar{y}_{dm}/y_d$  related to the envelope check by varying  $\alpha$  and  $n$  with measurement error equal to 0.05 (a) and 0.20 (b).

#### 5.4.3 APPLICABILITY AND LIMITATIONS OF THE OUTCOMES OF THE SAFETY ASSESSMENT CONSIDERING KNOWLEDGE UNCERTAINTIES

The presented outcomes of the safety assessment of existing PT concrete girders, evaluated through the semi-probabilistic approach encompassing

knowledge uncertainties can be analysed by highlighting their pros and some limitations.

For what concerns the applicability of the study, it is worth noting that the study confirms the general robustness of the semi-probabilistic Eurocodes safety format and the investigated range of both  $\alpha$  and  $n$ , since it always leads to a safe estimation of the safety ratio, with respect to a theoretically condition of perfect knowledge. The assumption of a confidence level  $\alpha=0.75$ , as recommended by the Eurocodes [3],[5], combined with a limited number of tests  $n$ , seems to be a good compromise to achieve reliable results and to limiting the economic efforts.

The measurement error provides the chance to account for a further source of uncertainties, and its impact on the results is significant since it leads to lower expected values of the safety ratio  $\bar{y}_{d,m}$ , with respect to those estimated without considering it. However, the values of the coefficient of variation of the measurement error need to be deeply investigated, since very limited information are available in the literature. This aspect may notably influence the outcomes, especially for what concerns tests and technologies for the estimation of the tensile force and strength of prestressing cable [7], as well as of the detrimental effects of corrosion [8], [9]. Moreover, the measurement error produces a considerable underestimation of the safety ratio, which in practice results in failed structural checks. This leads to unnecessary demolition and rebuilt cost, since the checks would be nearly satisfied in the case of perfect knowledge.

## 5.5 PROBABILISTIC ANALYSIS AND FAILURE PROBABILITY

In this section, the effect of knowledge uncertainties on probability of failure  $P_f$  is investigated. The probability of failure considered herein represents the expected annual rate that the effect of an applied action exceeds the system's strength. The bending and the shear failure are taken into account and considered independently.

Initially, the analysis extends beyond the basic random variables associated with strength to include other probabilistic factors, such as loads and model uncertainties. Subsequently, the two methodologies adopted to incorporate the impact of knowledge uncertainties into the probabilistic analysis are detailed.

### 5.5.1 BASIC RANDOM VARIABLE FOR RELIABILITY ASSESSMENT

In the previous results computed in terms of capacity/demand ratio only the strength random variables are considered and summarized in Table 5-1. Performing a probabilistic analysis, it is essential to consider additional basic random variables including those related to model uncertainties and loads.

The model uncertainties are considered for both the load effect and strength model. These uncertainties are function of the considered type of failure. Their values are summarized in Table 5-14 [10].

The loads are the permanent one, due to the self-weight of the structure, and the traffic load LM1, [1], having a time reference of 1 year. Their distribution are summarised in Table 5-15

*Table 5-14 Probability distributions of strength and load model uncertainties [10]*

| Basic variables          | Distribution type | $\mu$                      | $c_v$ |
|--------------------------|-------------------|----------------------------|-------|
| $\vartheta_R$<br>Bending | Log-Normal        | $\vartheta_{Em} = 1.2 [-]$ | 0.15  |
| $\vartheta_R$<br>Shear   | Log-Normal        | $\vartheta_{Em} = 1.2 [-]$ | 0.25  |
| $\vartheta_E$<br>Bending | Log-Normal        | $\vartheta_{Em} = 1 [-]$   | 0.10  |
| $\vartheta_E$<br>Shear   | Log-Normal        | $\vartheta_{Em} = 1 [-]$   | 0.10  |

*Table 5-15 Probability distributions of the basic variables of loads*

| Basic variables | Distribution type | $\mu$                 | $c_v$               |
|-----------------|-------------------|-----------------------|---------------------|
| G               | Log-Normal        | $G_m = G_{op}$<br>[3] | 0.05<br>[3]         |
| Q               | Gumbel            | $Q_m = t_{ref} 1y$    | 0.075<br>Table 3-12 |

### 5.5.2 PROBABILISTIC ANALYSIS TOOL

Several methods can be employed to estimate the failure probability of a response quantity of interest. Among these, Monte Carlo (MC), Section 2.5.3.1, simulation method offers a simple and robust means for estimating the failure probability. However, it is not computationally efficient for estimating small failure probabilities since the number of samples required to achieve a given accuracy is inversely proportional to the failure probability when it is small, i.e.  $10^{-6}$ . Approximate reliability methods, such as FORM and SORM, (Section 2.5.2), rely on identifying a design point to estimates the failure probability. However, their efficient decreases with an increasing number of variables involved and the degree of nonlinearity of the system response. Given the high dimension and complexity of the problem in this study, these methods are not suitable.

For these reasons, the Subset simulation technique, Section 2.5.3.2, is adopted.

### 5.5.3 METHODOLOGY

The probability of failure  $P_f$  can be computed by integrating the probability distribution of the limit state function. By incorporating the model uncertainties into Eq. (2-7), the limit state function is expressed as:

$$z(\mathbf{X}) = \vartheta_R \cdot r(\mathbf{X}) - \vartheta_E \cdot e(\mathbf{X}) \quad (5-2)$$

The uncertainties of knowledge process are evaluated using two approaches: a simplified and a simulation method, taken as reference method. In the first, the uncertainties of the knowledge process are injected as a multiplier  $k$  applied to the strength response.  $k$  is the ratio between the  $\bar{y}_m/y$  that were computed and illustrated in previous section. The simulation method employs instead MC simulation to determine the probability distribution of the possible estimates of the basic random variables related to the strength then used in the subset simulation  $P_f$ . The mean values of these distributions are calculated using Eq. (A-54) while their standard deviation is computed using Eq. (A-59).

### 5.5.4 PROBABILISTIC OUTCOMES

The probabilities of failure  $P_f$  in the perfect knowledge scenario, taken as reference case, are performed for both bending and shear failures condition using the simulation method. The results are:

- $P_{f,b} = 6 \cdot 10^{-6}$
- $P_{f,s} = 3.5 \cdot 10^{-5}$

Another time, the results are quite close to reliability level required for new structure. The results of  $P_f$  in the uncertainties knowledge scenario are shown in the following sections.

#### 5.5.4.1 OUTCOMES WITHOUT MEASUREMENT ERROR

This scenario simulates a potential experimental campaign conducted on the case study, assuming no measurement errors and consequently considering tests affected only by the randomness of the variable measured. The results obtained are presented in Table 5-16, Table 5-17 for bending probability of failure  $\bar{P}_{f,b}$ , and in Table 5-19 for shear one  $\bar{P}_{f,s}$  obtained for different values of  $n$  and  $\alpha$ . To improve the tables readability, the results, the results of Table 5-16 and Table 5-17 are shown in Figure 5-11, while the results from Table 5-18 and Table 5-19 are presented Figure 5-12.

For the bending failure, both the simplified method Figure 5-11 (a) and simulation method Figure 5-11 (b) yield higher values than the reference case across all the

$n - \alpha$  combinations. Notably, between the two methods, the simulation one provides results closer to the reference case, because it accounts the uncertainties of the standard deviation derived from the experimental campaign. By reducing  $n$ , the results are more far respect the reference case. Additionally, the choice of confidence level  $\alpha$  significantly impacts the results. As  $\alpha$  increases,  $\bar{P}_{f,b}$  deviates more substantially from the reference  $P_f$ , making the condition overly strict, especially for lower  $n$ . The results differ by up to two orders of magnitude. Similar trend is observed for shear check, as shown in Figure 5-12.

As in the results of the semi-probabilistic method, the assumption of a confidence level  $\alpha=0.75$  seems to be a good compromise to achieve reliable results and to limiting over-conservatism and consequently the economic efforts.

*Table 5-16 Failure probabilities  $P_{f,b}$  simplified method without measurement error*

| $P_{f,b}$ |    | $\alpha$            |                      |                   |
|-----------|----|---------------------|----------------------|-------------------|
|           |    | 0.75                | 0.85                 | 0.95              |
| $n$       | 3  | $1.2 \cdot 10^{-5}$ | $2.6 \cdot 10^{-5}$  | $2 \cdot 10^{-4}$ |
|           | 8  | $1.1 \cdot 10^{-5}$ | $1.53 \cdot 10^{-5}$ | $2 \cdot 10^{-5}$ |
|           | 25 | $7.9 \cdot 10^{-6}$ | $9 \cdot 10^{-6}$    | $9 \cdot 10^{-6}$ |

*Table 5-17 Failure probabilities  $P_{f,b}$  simulation method without measurement error*

| $P_{f,b}$ |    | $\alpha$            |                     |                     |
|-----------|----|---------------------|---------------------|---------------------|
|           |    | 0.75                | 0.85                | 0.95                |
| $n$       | 3  | $1.7 \cdot 10^{-5}$ | $2.9 \cdot 10^{-5}$ | $3.4 \cdot 10^{-4}$ |
|           | 8  | $9.6 \cdot 10^{-6}$ | $1.3 \cdot 10^{-5}$ | $1.7 \cdot 10^{-5}$ |
|           | 25 | $8 \cdot 10^{-6}$   | $8.2 \cdot 10^{-6}$ | $9.6 \cdot 10^{-6}$ |

*Table 5-18 Failure probabilities  $P_{f,s}$  simplified method without measurement error*

| $P_{f,s}$ |    | $\alpha$            |                     |                     |
|-----------|----|---------------------|---------------------|---------------------|
|           |    | 0.75                | 0.85                | 0.95                |
| $n$       | 3  | $1.4 \cdot 10^{-4}$ | $2.8 \cdot 10^{-4}$ | $4.5 \cdot 10^{-3}$ |
|           | 8  | $6.6 \cdot 10^{-5}$ | $8.5 \cdot 10^{-5}$ | $2 \cdot 10^{-4}$   |
|           | 25 | $4.7 \cdot 10^{-5}$ | $5.3 \cdot 10^{-5}$ | $7.5 \cdot 10^{-5}$ |

Table 5-19 Failure probabilities  $P_{f,s}$  simulation method without measurement error

| $P_{f,b}$ |    | $\alpha$            |                     |                     |
|-----------|----|---------------------|---------------------|---------------------|
|           |    | 0.75                | 0.85                | 0.95                |
| $n$       | 3  | $2.7 \cdot 10^{-4}$ | $1.4 \cdot 10^{-3}$ | $5 \cdot 10^{-3}$   |
|           | 8  | $6.7 \cdot 10^{-5}$ | $1.2 \cdot 10^{-4}$ | $3.2 \cdot 10^{-4}$ |
|           | 25 | $5.2 \cdot 10^{-5}$ | $6 \cdot 10^{-5}$   | $7.9 \cdot 10^{-5}$ |

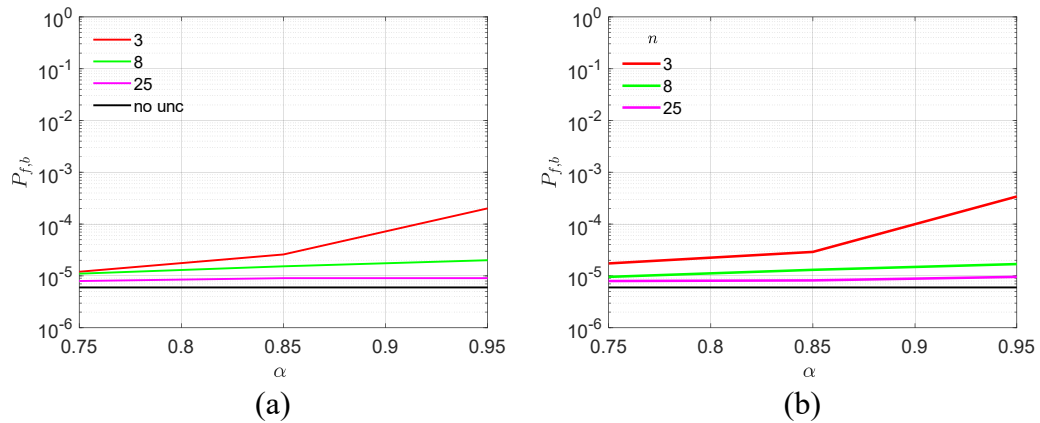


Figure 5-11 Failure probabilities  $P_{f,b}$  without the measurement error: (a) simplified approach, (b) simulation approach

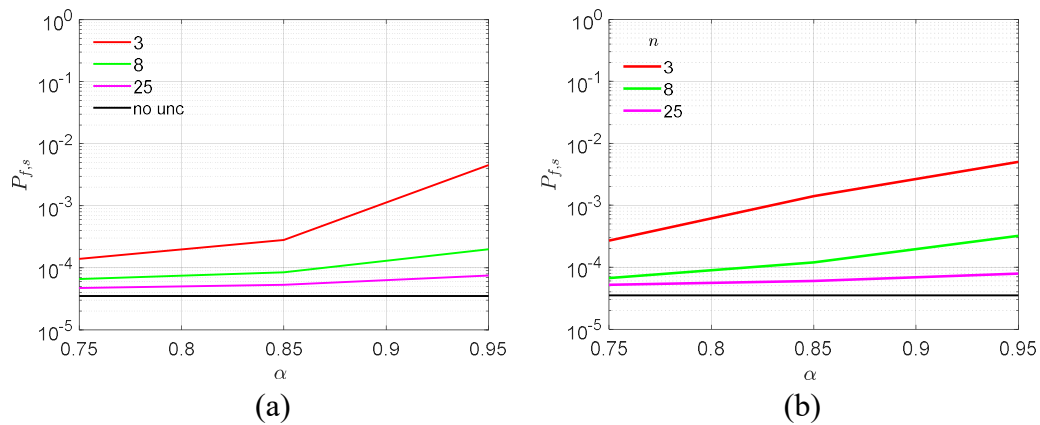


Figure 5-12 Failure probabilities  $P_{f,s}$  without the measurement error: (a) simplified approach, (b) simulation approach

#### 5.5.4.2 OUTCOMES WITH MEASUREMENT ERROR

In this scenario, measurement error is incorporated into the analysis. The results are showed in Figure 5-13 and Figure 5-14. In this scenario, results are affected by the intrinsic variability of the basic variables plus the uncertainties related to the measurement error, amplifying the global dispersion. Consequently, the same general considerations regarding  $n$  and  $\alpha$  from the previous case remain applicable but with a significant increase of the estimated probabilities, even for  $\alpha = 0.75$ . At this  $\alpha$  value, the simplified method shows differences up to one order of magnitude compared to the reference case. In contrast, the simulation method demonstrates deviations of up to three orders of magnitude when  $n = 3$ . This substantial discrepancy arises because the simulation method directly integrates measurement error into the probability distributions of the resistance variables, which significantly influences the estimated probability of failure. As  $\alpha$  increases, the divergence from reference values becomes even more pronounced.

The inclusion of measurement error introduces an additional layer of uncertainty, resulting in lower expected values compared to estimates that neglect this factor. However, the coefficient of variation associated with measurement error requires further investigation, including determining its actual value for different test types and developing effective strategies for its integration into probabilistic analyses.

Table 5-20 Failure probabilities  $P_{f,b}$  simplified method with measurement error ( $c_{v,\varepsilon} = 0.10$ )

| $P_{f,b}$ |    | $\alpha$            |                     |                     |
|-----------|----|---------------------|---------------------|---------------------|
|           |    | 0.75                | 0.85                | 0.95                |
| $n$       | 3  | $6.5 \cdot 10^{-4}$ | $3.5 \cdot 10^{-3}$ | $3.4 \cdot 10^{-2}$ |
|           | 8  | $2.1 \cdot 10^{-4}$ | $4.8 \cdot 10^{-4}$ | $2.2 \cdot 10^{-3}$ |
|           | 25 | $1 \cdot 10^{-4}$   | $1.6 \cdot 10^{-4}$ | $3.4 \cdot 10^{-4}$ |

Table 5-21 Failure probabilities  $P_{f,b}$  simulation method with measurement error ( $c_{v,\varepsilon} = 0.10$ )

| $P_{f,b}$ |    | $\alpha$            |                     |                     |
|-----------|----|---------------------|---------------------|---------------------|
|           |    | 0.75                | 0.85                | 0.95                |
| $n$       | 3  | $1.5 \cdot 10^{-2}$ | $6.6 \cdot 10^{-2}$ | 0.4                 |
|           | 8  | $1.2 \cdot 10^{-3}$ | $3.7 \cdot 10^{-3}$ | $1.6 \cdot 10^{-2}$ |
|           | 25 | $3.8 \cdot 10^{-4}$ | $6.5 \cdot 10^{-4}$ | $1.5 \cdot 10^{-3}$ |

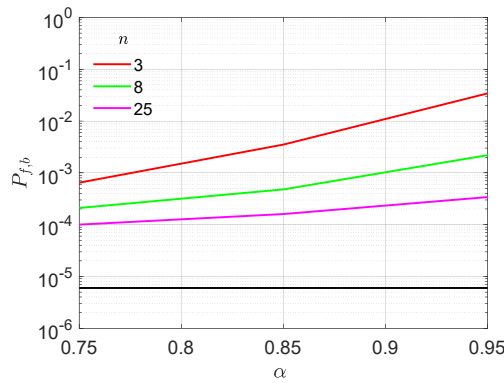
Table 5-22 Failure probabilities  $P_{f,s}$  simplified method with measurement error ( $c_{v,\varepsilon} = 0.10$ )

| $P_{f,s}$ |   | $\alpha$            |                     |                     |
|-----------|---|---------------------|---------------------|---------------------|
|           |   | 0.75                | 0.85                | 0.95                |
| $n$       | 3 | $9.4 \cdot 10^{-4}$ | $6.2 \cdot 10^{-3}$ | 0.13                |
|           | 8 | $1.4 \cdot 10^{-4}$ | $5.4 \cdot 10^{-4}$ | $2.7 \cdot 10^{-3}$ |

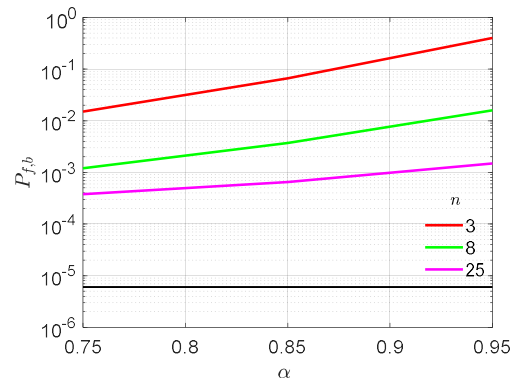
|           |                     |                     |                     |
|-----------|---------------------|---------------------|---------------------|
| <b>25</b> | $5.1 \cdot 10^{-5}$ | $1.6 \cdot 10^{-4}$ | $2.8 \cdot 10^{-4}$ |
|-----------|---------------------|---------------------|---------------------|

Table 5-23 Failure probabilities  $P_{f,s}$  simulation method with measurement error ( $c_{v,\varepsilon} = 0.10$ )

| $P_{f,s}$ |    | $\alpha$            |                     |                     |
|-----------|----|---------------------|---------------------|---------------------|
|           |    | 0.75                | 0.85                | 0.95                |
| $n$       | 3  | $1.8 \cdot 10^{-2}$ | $9.2 \cdot 10^{-2}$ | 0.4                 |
|           | 8  | $7.8 \cdot 10^{-4}$ | $2.4 \cdot 10^{-3}$ | $2 \cdot 10^{-2}$   |
|           | 25 | $2.1 \cdot 10^{-4}$ | $4.2 \cdot 10^{-4}$ | $1.1 \cdot 10^{-3}$ |

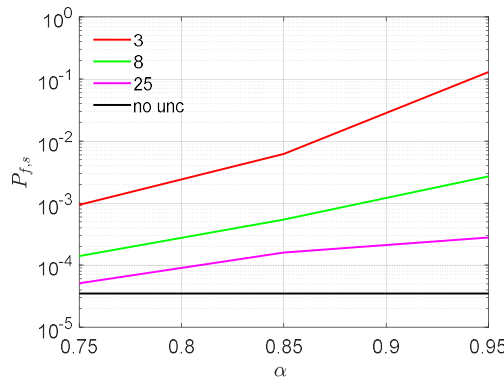


(a)

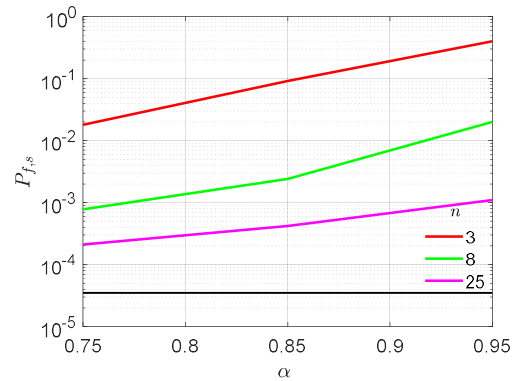


(b)

Figure 5-13 Failure probabilities  $P_{f,b}$  with the measurement error: (a) simplified approach, (b) simulation approach



(a)



(b)

Figure 5-14 Failure probabilities  $P_{f,s}$  with the measurement error: (a) simplified approach, (b) simulation approach

## 5.6 REFERENCES

- [1]CEN, 2005. EN 1991. Eurocode 1 – Actions on Structures, Brussels
- [2]CSI. SAP200. Computers and Structures Inc. Berkley, CA.
- [3]CEN, 2002. EN 1990 Eurocode 0- Basis of Structural Design., Brussels
- [4]CEN EN 1992-1 Eurocode 2- Design of Concrete Structures, Part1: CEN 2005. Brussels
- [5]Lemons, D.S.; Langevin, P., 2002. An Introduction to Stochastic Processes in Physics: Containing “On the Theory of Brownian Motion” by Paul Langevin, Translated by Anthony Gythiel; Johns Hopkins University Press: Baltimore, 2002; ISBN 978-0-8018-6866-5.
- [6] Leonardo da Vinci Pilot Project CZ/02/B/F/PP-134007, 2005 Handbook 2. Reliability Backgrounds, Guide to the basis of structural reliability and risk engineering related to Eurocodes, supplemented by practical example.
- [7]Morelli, F.; Panzera, I.; Piscini, A.; Salvatore, W.; Chichi, F.; Marconi, G.; Maestrini, D.; Gammino, M.; Mori, M., 2021. X-Ray Measure of Tensile Force in Post-Tensioned Steel Cables. Construction and Building Materials, 305, 124743, doi:[10.1016/j.conbuildmat.2021.124743](https://doi.org/10.1016/j.conbuildmat.2021.124743).
- [8]Ma, Y.; Zhang, J.; Wang, L.; Liu, Y., 2013. Probabilistic Prediction with Bayesian Updating for Strength Degradation of RC Bridge Beams. Structural Safety, 44, 102–109, doi:[10.1016/j.strusafe.2013.07.006](https://doi.org/10.1016/j.strusafe.2013.07.006).
- [9]Vereecken, E.; Lombaert, G.; Caspeele, R., 2021. Bayesian Assessment of the Remaining Prestressing Reinforcement in Existing Concrete Bridges. In fib symposium proceedings, 1386-1397
- [10]JCSS, 2002. Probabilistic Model Code, Part 3: Resistance Models, Joint Committee on Structural Safety.

# **6 TIME-DEPENDENT RELIABILITY AND DEGRADATION PHENOMENA**

## **6.1 INTRODUCTION**

The previous chapter thoroughly investigates the influence of knowledge uncertainties on the safety assessment of the existing post-tensioned concrete bridge, excluding the presence of degradation. This assumption does not fully reflect reality since existing structures are subjected to deterioration mechanisms that reduce their safety over time.

For this reason, this part of thesis investigates the impact of degradation on safety assessment of bridges. The focus is shifted from the evolution of the static behaviour of bridge PT deck to the performance of reinforced concrete bridge piers subjected to natural hazards to achieve a fully comprehensive assessment of bridge's safety. Specifically, bridge piers are subjected to several hazard that can lead to significant socio-economic impact on society [1] such as floods, landslides, manmade hazards, as well as the detrimental effects due to their combinations. However, the seismic risk remains one of the most critical concerns given the large number of structures located in seismic area, such as Italy, which faces a high seismic risk [2].

This chapter examines the combined effect of seismic hazard and structural degradation on bridge piers. Initially, it explains and describes the phenomena leading the reinforcement corrosion and their respective models. Moreover, the direct and indirect corrosion effects on material strength are then analysed introducing several empirical models recommended in the literature. Next, the methodology for evaluating the combination of seismic risk and corrosion is discussed and summarized.

## **6.2 CORROSION REINFORCING STEEL**

This section illustrates the phenomenon leading to the corrosion of reinforcement in reinforced concrete structures. The carbonation and chloride-induced corrosion are investigated. Models necessary for calculating the initiation time of corrosion, as well as those describing the time evolution of different types of corrosion (uniform and pitting), are introduced. In addition to directly reducing the cross-sectional area, corrosion also produces indirect effects, such as the

reduction of the mechanical properties of steel and concrete due to cracking. Various empirical models from the literature are presented.

### 6.2.1 INITIATION TIME

Most of all corrosion damage is caused by the neutralisation of the concrete through carbonation or the too high concentration of chloride surrounding the reinforcement. In the following the main models describing the corrosion initiation are reported.

#### CARBONATION-INDUCED CORROSION

The carbonation of concrete is a chemical reaction between the environmental carbon dioxide and the alkaline compounds within the concrete pores. If carbonation reaches the surface of the reinforcement bar, the pH of concrete decreases, leading to the loss of passivating layer of bar and triggering corrosion. This process is influenced by factor such as the relative humidity, drying and wetting of the concrete, concrete inhomogeneity, and other minor factors [1].

The following model, widely used in projects [1], [4] and codes[5], describes the diffusion of the carbonation depth  $x_c$  over the time  $t$  as follows

$$x_c(t) = \sqrt{2 \cdot k_e \cdot k_c \cdot (k_t \cdot R_{ACC,0}^{-1} + \varepsilon_t) \cdot C_s \cdot \sqrt{t} \cdot W(t)} \quad (6-1)$$

where  $k_e$  is the environmental function;  $k_c$  is the execution transfer parameter;  $k_t$  is the regression parameter;  $R_{ACC,0}^{-1}$  is the inverse effective carbonation resistance of concrete;  $\varepsilon_t$  is the error term;  $C_s$  is  $CO_2$  concentration content in the atmosphere;  $W(t)$  is the weather function. The carbonation-induced corrosion is triggered when  $x_c$  reaches the bars far from the concrete surface of a concrete cover  $c$ . The time  $t$  in which this occur is called initiation time  $T_i$ . In the following, the definition of some previous parameter is reported; their values can be found in [4], [5].

The environmental function  $k_e$  takes account of the influence of the humidity level on the diffusion of carbonation into the concrete. It can be described as follows

$$k_e = \left( \frac{1 - RH_{real}^{f_e}}{1 - RH_{ref}^{f_e}} \right)^{g_e} \quad (6-2)$$

where  $RH_{real}$  is the relative humidity of the environment;  $RH_{ref}$  is the reference humidity;  $f_e$  and  $g_e$  are exponent coefficients.

The execution transfer parameter  $k_c$  takes account the curing influences on the diffusion properties of the concrete and depends on the chosen material as well as

the environment and the curing condition. It can be determined using Eq. (6-3) in which  $t_c$  is the period of curing and  $b_c$  is a regression parameter.

$$k_c = \left(\frac{t_c}{7}\right)^{b_c} \quad (6-3)$$

For the model introduced above, the inverse effective carbonation resistance is determined by accelerated laboratory test (ACC). The relationship between the inverse carbonation resistances under natural conditions (NAC) is given by the Eq. (6-4). By using ACC there are some benefits such as short duration, binding capacity and changes of the carbonation resistance can be neglected. This procedure is constituted by indirect and direct tests.  $R_{ACC,0}^{-1}$  can be also deduced as function of cement type and water cement ratio.

$$R_{NAC,0}^{-1} = k_t \cdot R_{ACC,0}^{-1} + \varepsilon_t \quad (6-4)$$

The weather function  $W$ , Eq. (6-5), considers meso-climatic conditions due to wetting events of the concrete surface. In Eq. (6-5),  $ToW$  is the time of wetness,  $p_{SR}$  is the probability of driving rain and  $b_w$  is the exponent regression.

$$W(t) = \left(\frac{t_0}{t}\right)^{\frac{(p_{SR} \cdot ToW)^{b_w}}{2}} \quad (6-5)$$

The diffusive process can be described in probabilistic terms by treating the previous parameters as random variables. Their distribution values are provided in [4], [5].

#### CHLORIDES-INDUCED CORROSION

Chlorides are a critical issue for huge stock of existing reinforced concrete (r.c.) structures [7] and are driven by concentration gradients inside the material volume [8]. Chlorides can derive from the environment, and they diffuse into concrete as airborne and/or by direct contact with seawater, particularly in tidal and splash zone. Additionally, the use of de-icing salts on bridge decks, for snow and ice removal, is another source of chloride penetration [9]. Their diffusion is influenced by the environmental exposure conditions, structural aging and material composition of the concrete.

The diffusive process of chloride concentration is described by the Fick's model [5]. The penetration profile of chlorides is estimated adopting the following equation [5]

$$C(x, t) = C_0 + (C_{S,\Delta x} - C_0) \cdot \left[ 1 - \operatorname{erf} \left( \frac{c - \Delta x}{2 \cdot \sqrt{D_{app}(t) \cdot t}} \right) \right] \quad (6-6)$$

where  $C(x, t)$  is the chloride concentration at the depth  $x$  at the time  $t$ ;  $C_0$  is the initial chloride content in the cement post;  $\Delta x$  is the depth of the convection zone;  $C_{S,\Delta x}$  is the chloride content at the depth  $\Delta x$ ;  $\operatorname{erf}$  is the Gaussian error function;  $D_{app}$  is the apparent coefficient of chloride diffusion through concrete that takes into account the reduction of diffusion over the time.  $D_{app}$  can be evaluated as follows

$$D_{app}(t) = k_e \cdot D_{c0} \cdot \left( \frac{t_0}{t} \right)^\alpha \quad (6-7)$$

where  $D_{c0}$  is the initial value of chloride diffusion coefficient at the reference time  $t_0$ ;  $t_0$  is the reference time equal to 28 days;  $\alpha$  is the aging exponent;  $k_e$  is the environmental variable that takes into consideration the effect of temperature on chloride ingress in concrete and is expressed in Eq. (6-8).

$$k_e = \exp \left( b_e \cdot \left( \frac{1}{T_{ref}} - \frac{1}{T_{real}} \right) \right) \quad (6-8)$$

In the previous equation,  $b_e$ ,  $T_{ref}$ ,  $T_{real}$  represent respectively temperature coefficient, the reference temperature and the temperature of the structural element or the ambient air.

The chloride corrosion is triggered when the  $C(x, t)$  exceeds a fixed threshold denoted as the critical chloride concentration  $C_{crit}$ . The time  $t$  in which this occur is called initiation time  $T_i$ .

In according to Fick's diffusion model the initiation time  $T_i$  can be calculated directly as follows:

$$T_i = \frac{(c - \Delta x)^2}{4 \cdot D_{app}} \cdot \left[ \operatorname{erf}^{-1} \left( \frac{C_{S,\Delta x} - C_{crit}}{C_{S,\Delta x} - C_0} \right) \right]^{-2} \quad (6-9)$$

where this approach is assumed that the coefficient of chloride diffusion does not varies over the time [10] and equal to its initial value  $D_{app} = D_{c0}$ . This assumption is valid for short exposure time intervals.

The diffusive process can be described in probabilistic terms by treating the previous parameters as random variables. Their distribution values are function of the exposure classes, the cement types and the cement water ratio. Recommendations are provided in [5] and in [11]; the latter gives specific values for several countries in the worldwide.

### 6.2.2 CORROSION RATE

The period during which the corrosion is actively is called propagation time  $t_p = t - T_i$ . The main parameter describing the process is the annual corrosion rate  $V_{corr}$  [ $\mu\text{m}/\text{year}$ ] that can be expressed in terms of the current density  $i_{corr}$  [ $\mu\text{A}/\text{cm}^2$ ] as it is given by [4]

$$V_{corr} = 11.6 \cdot i_{corr} \quad (6-10)$$

The corrosion rate can vary over the time, and it is influenced by:

- temperature:  $V_{corr}$  increases as the temperature rises.
- concrete humidity: following rain events,  $V_{corr}$  increases.
- concrete resistivity:  $V_{corr}$  increases as the water accumulate in the concrete pores.

Moreover,  $V_{corr}$  assumes different values for the two phenomena showed above:

- in the case of carbonation-induced corrosion,  $V_{corr}$  increases with rising of the relative humidity and it becomes significant only for high humidity levels. However, its values are always lower than chloride-induced corrosion.
- in the case of chloride-induced corrosion, the chloride concentration derived from environment influences the  $V_{corr}$ .

Like the parameter of initiation time,  $V_{corr}$  and  $i_{corr}$  can also treated as random variables.

Several methods, proposed in literature, estimate  $V_{corr}$  from  $i_{corr}$ , which can be directly deduced through direct measurement both for new structures with laboratory tests and for existing structures with linear polarization technique [4]. Other documents propose empirical methods ([1], [4], [12]). The uncertainties regarding these methods are available in the cited literature. Additionally,  $i_{corr}$  values are also available based on exposure classes as provided in [1] and are summarized in Table 6-1 for carbonation-induced corrosion and in Table 6-2 for chloride-induced one.

Table 6-1  $i_{corr}$  distribution for carbonation-induced corrosion [1]

| Exposure class               | Distribution type | $\mu$                               | $\sigma$                            |
|------------------------------|-------------------|-------------------------------------|-------------------------------------|
| Dry                          | Lognormal         | 0                                   | 0                                   |
| Wet rarely dry (unsheltered) | Lognormal         | 0.345 [ $\mu\text{A}/\text{cm}^2$ ] | 0.259 [ $\mu\text{A}/\text{cm}^2$ ] |

|                               |           |                        |                        |
|-------------------------------|-----------|------------------------|------------------------|
| Moderate humidity (sheltered) | Lognormal | 0.172 [ $\mu A/cm^2$ ] | 0.086 [ $\mu A/cm^2$ ] |
| Cyclic wet-dry (unsheltered)  | Lognormal | 0.431 [ $\mu A/cm^2$ ] | 0.259 [ $\mu A/cm^2$ ] |

Table 6-2  $i_{corr}$  distribution for chloride-induced corrosion [1]

| Exposure class     | Distribution type | $\mu$                  | $\sigma$               |
|--------------------|-------------------|------------------------|------------------------|
| Wet-rarely dry     | Lognormal         | 0.345 [ $\mu A/cm^2$ ] | 0.259 [ $\mu A/cm^2$ ] |
| Cyclic wet-dry     | Lognormal         | 2.586 [ $\mu A/cm^2$ ] | 1.724 [ $\mu A/cm^2$ ] |
| Airborne sea water | Lognormal         | 2.586 [ $\mu A/cm^2$ ] | 1.724 [ $\mu A/cm^2$ ] |
| Submerged          | Lognormal         | Not expected           |                        |
| Tidal zone         | Lognormal         | 6.035 [ $\mu A/cm^2$ ] | 3.448 [ $\mu A/cm^2$ ] |

The corrosion rate may be directly given as [4]

$$V_{corr} = V_{corr,\alpha} \cdot ToW \quad (6-11)$$

where  $ToW$  is the wetness period given as the fraction of the year,  $V_{corr,\alpha}$  is the mean corrosion rate when the corrosion is active. The parameters are summarized in Table 6-3 for carbonation-induced corrosion and in Table 6-4 for chloride-induced one.

Table 6-3 Distribution of  $V_{corr,\alpha}$  and  $ToW$  for carbonation-induced corrosion [4]

|                   | Exposure class | Distribution type | $\mu$            | $\sigma$         |
|-------------------|----------------|-------------------|------------------|------------------|
| $V_{corr,\alpha}$ | Sheltered      | Weibull           | 0.002 [ $mm/y$ ] | 0.003 [ $mm/y$ ] |
| $V_{corr,\alpha}$ | Unsheltered    | Weibull           | 0.005 [ $mm/y$ ] | 0.007 [ $mm/y$ ] |
| $ToW$             | Sheltered      | Normal            | 0.50 [—]         | 0.12 [—]         |
| $ToW$             | Unsheltered    | Normal            | 0.75 [—]         | 0.20 [—]         |

Table 6-4 Distribution of  $V_{corr,\alpha}$  and  $ToW$  for chloride-induced corrosion [4]

|                   | Exposure class | Distribution type | $\mu$            | $\sigma$         |
|-------------------|----------------|-------------------|------------------|------------------|
| $V_{corr,\alpha}$ | Wet-rarely dry | Weibull           | 0.004 [ $mm/y$ ] | 0.006 [ $mm/y$ ] |

|                   |                    |         |              |              |
|-------------------|--------------------|---------|--------------|--------------|
| $V_{corr,\alpha}$ | Cyclic wet-dry     | Weibull | 0.030 [mm/y] | 0.040 [mm/y] |
| $V_{corr,\alpha}$ | Airborne sea water | Weibull | 0.030 [mm/y] | 0.040 [mm/y] |
| $V_{corr,\alpha}$ | Submerged          | Weibull | Not expected |              |
| $V_{corr,\alpha}$ | Tidal zone         | Weibull | 0.070 [mm/y] | 0.070 [mm/y] |
| $ToW$             | Wet-rarely dry     | Normal  | 1 [—]        | 0.25 [—]     |
| $ToW$             | Cyclic wet-dry     | Normal  | 0.75 [—]     | 0.20 [—]     |
| $ToW$             | Airborne sea water | Normal  | 0.50 [—]     | 0.12 [—]     |
| $ToW$             | Submerged          | Normal  | Not expected |              |
| $ToW$             | Tidal zone         | Normal  | 1 [—]        | 0.25 [—]     |

The recommended distribution, proposed in [1] and [4] as a function of the exposure class, results in different values of  $V_{corr}$ . Figure 6-1 shows the histograms of  $V_{corr}$  generated from the probability distribution for chlorides-induced corrosion (Table 6-2, Table 6-4). The histograms in Figure 6-1 (a) and Figure 6-1 (b) differ in shape due to the starting probability distributions. Additionally, the results exhibit different mean  $\mu$  and coefficient of variation  $c_v$  values, as reported in Table 6-5.

From a probability analysis perspective, another key difference is the number of distributions required to characterize  $V_{corr}$ . In this regard, the direct distribution proposed in [1] aims to minimize the number of problem variables, simplifying the analysis efficiency.

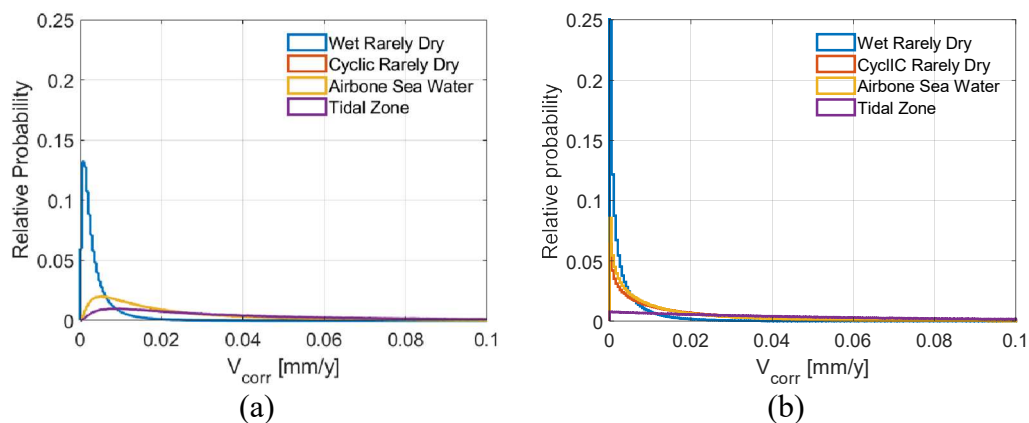


Figure 6-1 Histograms of  $V_{corr}$  for different exposure classes from recommendation given in [1] (a), and in [4] (b)

Table 6-5 Resulting values of  $\mu$  and  $c_v$  for the two approaches

| Exposure Class     | $V_{corr}$    |       |              |       |
|--------------------|---------------|-------|--------------|-------|
|                    | Duracrete [1] |       | LifeCon [4]  |       |
|                    | $\mu$ [mm/y]  | $c_v$ | $\mu$ [mm/y] | $c_v$ |
| Wet-rarely dry     | 0.004         | 1.33  | 0.004        | 1.57  |
| Cyclic wet-dry     | 0.03          | 1.50  | 0.023        | 1.41  |
| Airborne sea water | 0.03          | 1.5   | 0.015        | 1.40  |
| Submerged          | Not expected  |       | Not expected |       |
| Tidal zone         | 0.07          | 1.77  | 0.07         | 1.06  |

### 6.2.3 CORROSION EFFECT ON REINFORCEMENT

This section describes the influences of corrosion on concrete structures. Corrosion directly reduces the steel cross-section and can be distinguish between uniform and pitting corrosion. Additionally, corrosion indirectly affects the mechanical properties of steel by decreasing the strength and ultimate strain. Besides the influences on steel, there might also be a reduction of concrete strength and spalling of the concrete cover due to the concrete cracking induced by the expansive properties of rust. For all these influences, analytical and empirical models are discussed.

#### UNIFORM CORROSION

The uniform corrosion consists of uniform loss of metal cover over the whole exposed surface of a reinforcing bar. The reduction of diameter  $D(t)$  is expressed following equation [13]

$$D(t) = D_0 - 2 \cdot t_p \cdot V_{corr} \quad (6-12)$$

where  $D_0$  is the initial diameter,  $2 \cdot t_p \cdot V_{corr}$  is the corrosion penetration considered in both side of the bar.

The reduction of cross-sectional area of reinforcing steel  $A(t)$  is then equal to

$$A(t) = \begin{cases} \frac{\pi D_0^2}{4}, & t < T_i \\ \frac{\pi D(t)^2}{4}, & t > T_i \end{cases} \quad (6-13)$$

## PITTING CORROSION

Pitting corrosion is invariably associated with chloride, and it is a form of localized corrosion. When the corrosion is localized, the reduction in cross-sectional area is not uniform, but concentrated at the pit, as shown in Figure 6-2. In this figure,  $p(t)$  is the radius of the pit or also called pit depth at time  $t$ . Unlike uniform corrosion, the localized corrosion must be evaluated using different approach.

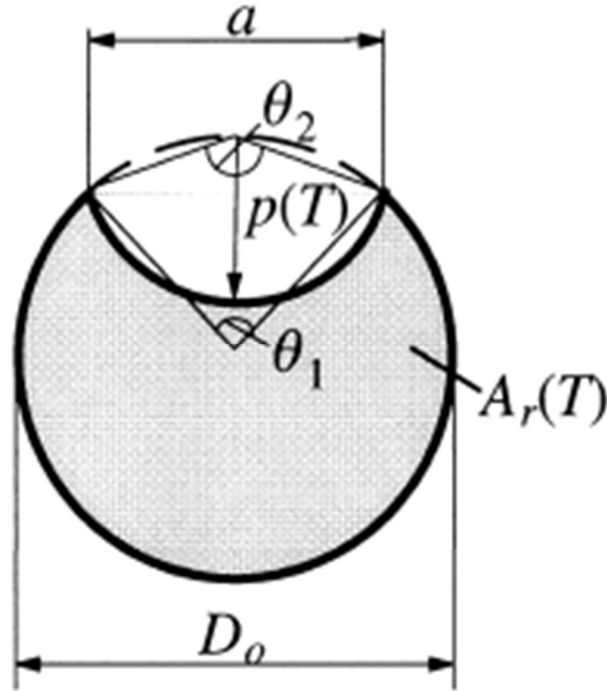


Figure 6-2 Pit configuration

The area of corroded bar at time  $t$  is given by the following equation [13]

$$A(t) = \begin{cases} \frac{\pi D_0^2}{4} - A_1 - A_2, & p(t) \leq \frac{\sqrt{2}}{2} D_0 \\ A_1 - A_2, & \frac{\sqrt{2}}{2} D_0 < p(t) \leq D_0 \\ 0, & p(t) > D_0 \end{cases} \quad (6-14)$$

The other parameter in Eq. (6-14) are given as follows

$$A_1 = \frac{1}{2} \left[ \theta_1 \left( \frac{D_0}{2} \right)^2 - a \left| \frac{D_0}{2} - \frac{p(t)^2}{D_0} \right| \right] \quad (6-15)$$

$$A_2 = \frac{1}{2} \left[ \theta_2 p(t)^2 - a \frac{p(t)^2}{D_0} \right] \quad (6-16)$$

$$a = 2p(t) \sqrt{1 - \left(\frac{p(t)}{D_0}\right)^2} \quad (6-17)$$

$$\theta_1 = 2\arcsin\left(\frac{2a}{D_0}\right) \quad (6-18)$$

$$\theta_2 = 2\arcsin\left(\frac{a}{p(t)}\right) \quad (6-19)$$

The pit depth  $p(t)$  can be estimated as

$$p(t) = V_{corr} \cdot t_p \cdot R \quad (6-20)$$

where  $R$  is the penetration ratio between the maximum and average penetration.

In the literature several methods and recommendations are available for determining the  $R$  value. In [14], the  $R$  value has been obtained through both natural corrosion and accelerated corrosion tests on steel bars embedded in concrete containing chlorides. For natural corrosion tests,  $R$  varies between the range from 4 to 8, whereas for accelerated corrosion tests, it ranges from 5 and 13. In this contribution  $R$  is deterministic. These results highlight that pitting corrosion is significantly more severe than uniform corrosion.

Reference [1] provides  $R$  values for both the carbonation-induced corrosion and chloride-induced corrosion, treating the latter as a random variable having a Log-Normal distribution. The recommendations from this study are summarized in Table 6-6.

Table 6-6 Penetration ratio  $R$  [1]

| Environment | Distribution type | $\mu$    | $\sigma$ |
|-------------|-------------------|----------|----------|
| Carbonation | Deterministic     | 2 [–]    | –        |
| Chloride    | Log-Normal        | 9.28 [–] | 4.04 [–] |

Following corrosion chloride tests on steel bar with different diameters, reference [15] proposes a Gumbel distribution for the penetration ratio  $R$ . This model also includes a relationship between  $R$  and the length of the reinforcing bar where the pit can occur. Although this approach is restricted to only bar diameters tested. The information about the parameters of  $R$  distribution can be found in the cited reference.

Reference [15] applies a truncated normal distribution for penetration ratio  $R$  with a mean value of 3 and  $c_v$  equal to 0.33 (truncated at 1), which is based on the assumption that the 95% percentile of  $R$  corresponds to 4.

In the literature, there is a uniform corrosion model, proposed in [18], including also the pitting (Figure 6-3). The area reduction is expressed as follows

$$\Delta A = \frac{\pi}{4} (2 \cdot p(t) \cdot D_0 - p(t)^2) \quad (6-21)$$

This combined model is not suitable for describing real cases of corrosion but is more applicable to laboratory tests with accelerated corrosion [19], [20].

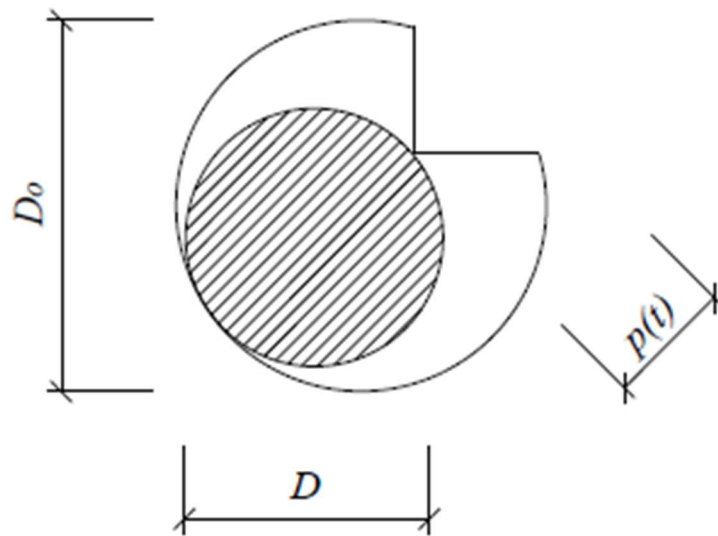


Figure 6-3 Cross-sectional reduction due to uniform and pitting corrosion

#### EFFECT ON DUCTILITY

The pitting corrosion affects also the ductility of steel reinforcement that can lead to a brittle failure of the concrete members [15], [21]. This behaviour can be observed in several experimental tensile tests on corroded bar collected in the literature. Figure 6-4 shows the load-elongation curves of a steel bars characterized by different levels of corrosion [22]. These results show that the bar exhibits brittle behaviour as corrosion increases. These results are confirmed by those obtained in [23], [24] in which the bars subjected to pitting corrosion exhibit significant loss of ductility of the reinforcement bar.

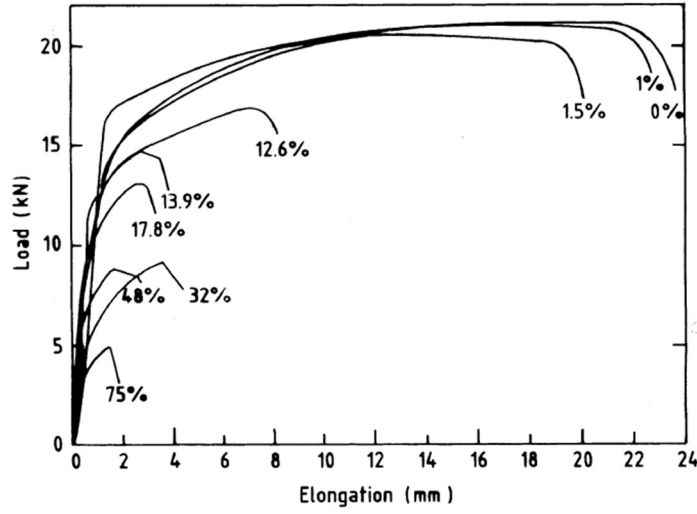


Figure 6-4 Long-elongation curves for steel bars with different degree of corrosion [22]

To model this behaviour an empirical approach is the one proposed by [21]. This approach introduces a linear reduction to describe the evolution of the ultimate strain of the steel, from  $\varepsilon'_{su} = \varepsilon_{su}$  in the not corroded material (percentage of the reduction of the bar cross section  $\alpha_{pit}$  equal to 0) to  $\varepsilon'_{su} = \varepsilon_{sy}$  ( $\alpha_{pit} = \alpha_{pit,max}$ ) for a complete loss of ductility. The function is defined as follows

$$\varepsilon'_{su} = \varepsilon_{sy} + (\varepsilon_{su} - \varepsilon_{sy}) \cdot \left(1 - \frac{\alpha_{pit}}{\alpha_{pit,max}}\right) \quad (6-22)$$

Where  $\alpha_{pit}$  is the ratio between the area of the pit and the initial area of the bar;  $\alpha_{pit,max}$  is a parameter describing the bar ductility. The application of this method is linked on the  $\alpha_{pit,max}$  choice. Different values of  $\alpha_{pit,max}$  have been measured by various authors and it ranges from 0.5 to 0.1 [21].

A manageable empirical formula, contained in [25], [26], defines the variation of the steel ultimate strain of the steel  $\varepsilon_{su}$  as follows

$$\varepsilon_{su} = \begin{cases} \varepsilon_{su0} & , 0 \leq \delta_s < 0.016 \\ 0.1521 \cdot \delta_s^{-0.4583} \cdot \varepsilon_{su0} & , 0.016 \leq \delta_s \leq 1 \end{cases} \quad (6-23)$$

in which  $\varepsilon_{su}$  is the nominal value of the steel ultimate strain,  $\delta_s$  is the cross-section reduction that depends on the dimensionless corrosion penetration index  $\delta$  as follows

$$\delta_s = \delta \cdot (2 - \delta) \quad (6-24)$$

The corrosion penetration index in the case of pitting corrosion is the ratio between the pitting penetration  $p$  and the un-corroded diameter of the bar.

Reference [27] presents also empirical correlation Eq. (6-25) linking the ultimate strength  $\varepsilon_{su}$  and the corrosion level  $Q_{corr}$  (ratio between the corroded and the initial area) through experimental coefficient  $\beta$ . Reference [27] suggests  $\beta$  value equal to 0.039. Other empirical coefficients, derived from other experimental works, are summarized in [28].

$$\varepsilon_{su} = (1 - \beta \cdot Q_{corr}) \cdot \varepsilon_{su0} \quad (6-25)$$

#### EFFECT ON STRENGTH PROPERTIES

The corrosion leads to an indirect effect of strength reduction of the steel bar. However, this effect can be neglected as it is highlighted in the research works presented in [23]. The reference [27] declares as executing salts spray tests on the corroded cross-section of the bars the yield strength and the ultimate strength partially varies following a linear behaviour. This work proposes a linear relationship (Eq. (6-26)) between the damage and the yield or the ultimate strength.

$$f = (1 - \beta \cdot Q_{corr}) \cdot f_0 \quad (6-26)$$

In the previous equation  $f_0$  is the initial resistance. The experimental coefficient  $\beta$  is suggested equal to 0.005 for both yield and ultimate strength [27]. Reference [4] highlights that the strength reduction of reinforcement bar is essentially due to the pitting corrosion, and it is a local phenomenon. Other empirical coefficients, derived from other experimental works, are summarized in [28].

#### CORROSION EFFECT ON CONCRETE

The corrosion produces an indirect degradation of the concrete. The increase of a bar volume given by the rust generate tensile strain in the concrete surrounding the bar. Once the strain strength of the concrete is exceeded, cracking develops and if the corrosion continues, the spalling of the concrete cover happens. The factors influencing the cracking of the concrete cover are the ratio between the concrete cover and the bar diameter  $c/d$  and the concrete quality. Higher  $c/d$  ratio delays the cracking while concrete characterizing with high water cement ratio allows to corrosion products to penetrate inside the pores. The cracking development even produces a reduction of compressive strength of concrete [29]. The reduced concrete strength can be calculated as follows [21]

$$f_{c,rid} = \frac{f_c}{1 + K \varepsilon_t / \varepsilon_{c0}} \quad (6-27)$$

where  $K$  is the coefficient related to the bar roughness and diameter (for medium-diameter ribbed bars  $K$  is equal to 0.1);  $\varepsilon_{c0}$  is the peak at the compressive stress  $f_c$ ;  $\varepsilon_t$  is the average tensile strain in the cracked concrete at the direction of the applied compression.  $\varepsilon_t$  can be calculated approximatively according to

$$\varepsilon_t = \frac{n_{bars} \cdot w}{b_i} \quad (6-28)$$

where  $n_{bars}$  is the bars number in the top layer (compressed bars);  $b_i$  is the section width without corrosion cracks;  $w$  is the crack width with a given corrosion level. Substituting  $\varepsilon_t$  in Eq. (6-27) it can be noticed that  $w$  is the unique variable governing the reduction of compressive strength.

Several methods are available in the literature about the crack width  $w$ . Following the approach proposed in [1] for uniform corrosion  $w$  is estimated as follows

$$w(t) = \begin{cases} 0 & , x < x_0 \\ w_0 + \beta \cdot (x - x_0) & , x > x_0 \end{cases} \quad (6-29)$$

where  $w_0$  is the initial cracking equal to  $0.05 \text{ mm}$ ;  $\beta$  is coefficient depending in the bar position ( $0.01$  for top cast bars and  $0.0125$  for bottom cast bars);  $x_0$  is the penetration attack necessary to produce a crack of width to  $0.005$ ;  $x$  is the actual penetration attack.  $x_0$  is evaluated as follows

$$x_0 = a_1 + a_2 \frac{c}{D_0} + a_3 f_{c,sp} \quad (6-30)$$

where  $c$  is the concrete cover;  $D_0$  is the initial diameter;  $a_1$ ,  $a_2$  and  $a_3$  are experimental parameters equal  $74.4 \text{ }\mu\text{m}$ ,  $7.3 \text{ }\mu\text{m}$  and  $-17.4 \text{ }\mu\text{m mm}^2/\text{N}$ ;  $f_{c,sp}$  is the characteristic tensile strength at the splitting given by reducing the characteristic tensile strength for 10%. According to [1], the spalling of the concrete cover happens when  $w(t)$  reaches  $1 \text{ mm}$ .

A more appropriate approach for experimental data is given by [30], which proposes a linear relationship between the width crack and the reduction of the cross-sectional bar Eq. (6-31). In this equation,  $K = 0.0575$  is a regression parameter based on experimental results,  $\Delta A_s$  is the reduction of bar section and  $\Delta A_{s0}$ , defined in Eq. (6-32), is function of the concrete cover  $c$ , initial bar diameter  $D_0$  and the pitting factor  $R$ . In the case of uniform corrosion,  $R$  in Eq. (6-32) is equal to 1.

$$w = K(\Delta A_s + \Delta A_{s0}) \quad (6-31)$$

$$\Delta A_{s0} = A_s \left[ 1 - \left[ 1 - \frac{R}{D_0} \left( 7.53 + 9.32 \frac{c}{D_0} \right) 10^{-3} \right]^2 \right] \quad (6-32)$$

Reference [21] introduces a relation to compute  $w$  Eq. (6-33) in which the rust is considered incompressible and uniform around the bar. In this equation,  $v_{rs}$  (equal to 2) is the ratio of the volumetric expansion of the oxides with respect to the virgin material and  $x$  is the penetration of the corrosion attack.

$$w = 2\pi(v_{rs} - 1)x \quad (6-33)$$

#### 6.2.4 COMPARISON AMONG METHODS

The previous section discusses the effects of corrosion on reinforcement bars, addressing both its direct impact (reduction of bar's cross-section) and its undirect effects, such as the concrete cracking or the bar's decreased ductility. The development of these effects over time has been analysed and modelled through several empirical methods. Therefore, this section aims to examine the differences among these methods to integrate them in a general model.

##### CORROSION EFFECT ON BAR AREA

The corrosion mechanism depends on the environment. The reduction of cross section can be uniform or localized in the case of pitting. The latter is most likely in chloride environment. Their different effects are herein analysed.

A  $\phi 18$  reinforcement bar is analysed for a period of propagation corrosion of 50 years. The uniform and pitting corrosion penetration are provided in Eq. (6-12) and Eq. (6-20) respectively. The area reduction of both corrosions are provided in Eq. (6-14) and Eq. (6-15). The corrosion rate  $V_{\text{corr}}$  is assumed for an airborne sea water environment and provided in Table 6-2. Figure 6-5 shows the deterministic ratio between the corrosion penetration  $x_t$  and the un-corroded diameter  $D_0$  over the time for uniform corrosion (green line) and the pitting one with difference penetration  $R$  proposed in literature. The pitting penetration is more aggressive respect the uniform; this phenomenon is more emphasized for high values of  $R$  for highly corrosion environment. The  $x_t$  produces a variation of diameter from which the area variation follows. Considering the previous results, Figure 6-6 shows the deterministic evolution of the ratio of the time-varying steel bar to the initial steel bar area  $A_{st}/A_{s0}$ . The results show that for high value of  $R$  the bar is completely corroded after 50 years, while the uniform corrosion (green line) leads to a loss of 40% of the initial area  $A_{s0}$ . Due to the geometrical model for pitting corrosion (Figure 6-2), the area losses for lower value of  $R$  (blue and orange lines) are lower respect the uniform corrosion for the propagation time considered.

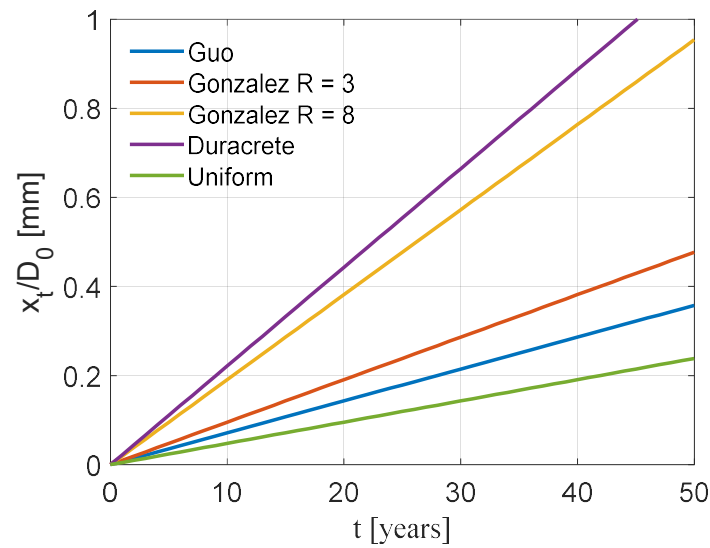


Figure 6-5 Deterministic penetration of  $x_t/D_0$  ratio for uniform corrosion (green line) and pitting corrosion

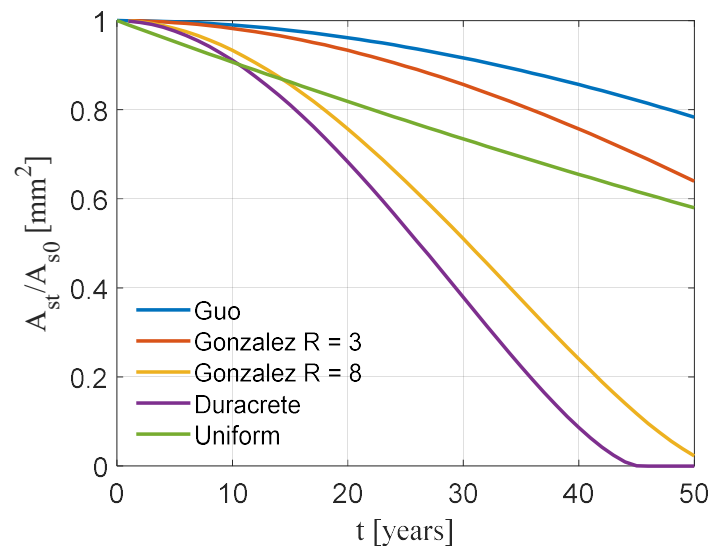


Figure 6-6 Deterministic evolution of  $A_{st}/A_{s0}$

#### DEGRADATION OF MECHANICAL PROPERTIES

Localized corrosion (pitting) reduces both the strength and ductility of reinforcement steel. While the reduction in strength can be neglected [23], experimental studies confirm a significant decrease in ductility of bars due to this type of corrosion. Figure 6-7 present an analyse of the two previously described method using pit depth values corresponding to an R value proposed in [1] (violet line in Figure 6-5). For the uncorroded case, the ultimate  $\varepsilon_{su0}$  and yield  $\varepsilon_{sy}$  strain of the steel are 0.05 and 0.00168 respectively. The two methods yield different results. The method, proposed in [21] and based on Eq. (6-22), (blue and brown

lines) is sensitive to the pit area and is greatly conditioned by the chosen  $\alpha_{pit,max}$ , which is challenging to estimate experimentally. On the contrary, the method shown in Eq. (6-23) and proposed in [25], (purple line) establishes a non-linear relation with the cross-section reduction  $\delta_s$ , which depends on corrosion pit depth. Moreover, this approach is simpler and has been validated through experimental data [26].

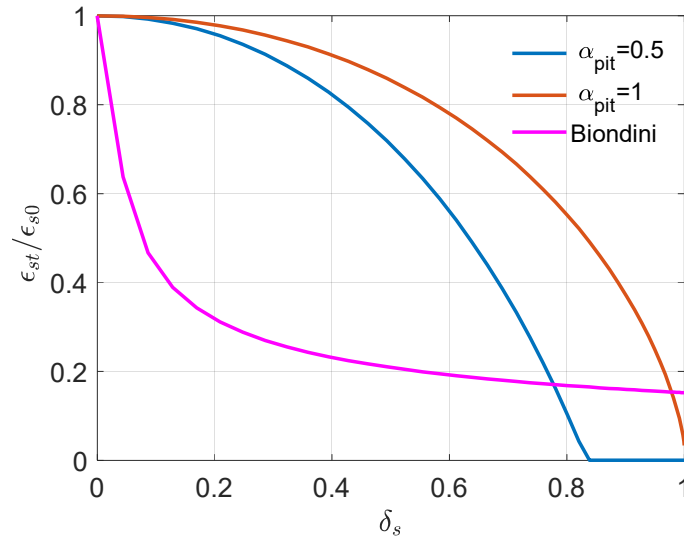


Figure 6-7 Models for ultimate strength variation: [21] blue and orange lines and [25] violet line

#### CONCRETE DEGRADATIONS

The development of concrete crack is estimated using empirical methods described in Section 6.2.3. The Duracrete [1] and Molina [21] models consider a uniform distribution of corrosion products around the reinforcing bar. To compare these three methods, the penetration depth results of uniform corrosion, shown in Figure 6-5, are adopted assuming a concrete cover  $c = 30 \text{ mm}$ . The fundamental differences between these models, illustrated in Figure 6-8, lie in their approach to crack formation. The Duracrete and Molina models consider the cracking as a function of the corrosion penetration, whereas Vidal [30] proposes a model in which crack width is related to the reduction of reinforcement area. This difference significantly influences crack formation and, consequently, the reduction in concrete compressive strength. In the following results, the concrete spalling is not considered and  $w$  is calculated over the recommended value of  $1 \text{ mm}$ .

Based on the results of  $w$  in Figure 6-8, the corresponding reduction of concrete strength is shown in Figure 6-10. The analysis assumes a rectangular section with a width of  $b_i = 300 \text{ mm}$ , 4 reinforcement bars at the bottom, an initial concrete strength  $f_{c0} = 42 \text{ MPa}$  and a peak at the compressive stress  $\epsilon_{c0} = 0.0022$ .

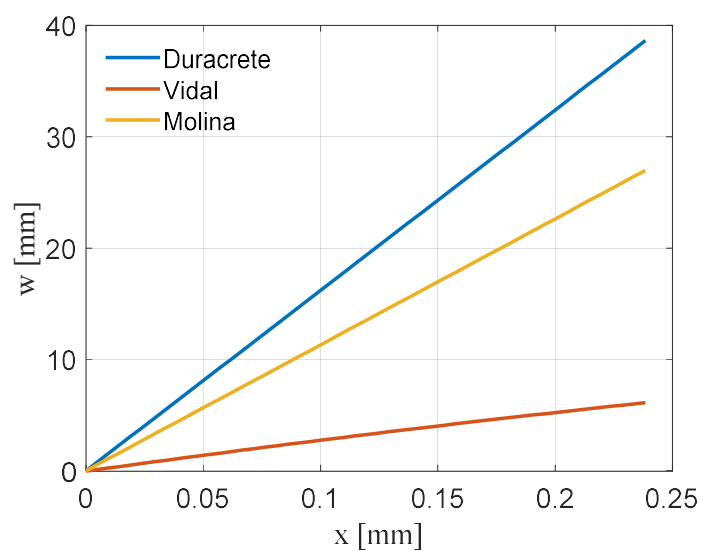


Figure 6-8 Comparison among crack width methods for  $\phi 18$

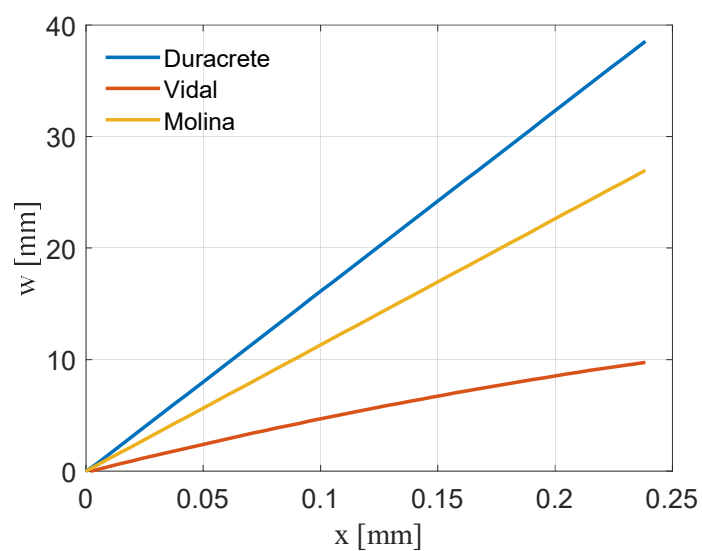


Figure 6-9 Comparison among crack width methods for  $\phi 10$

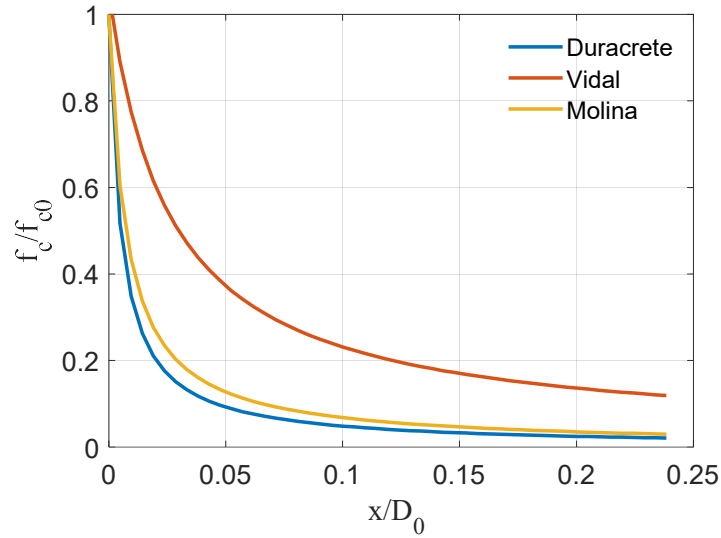


Figure 6-10 Comparison among the concrete strength reduction

### 6.3 METHODOLOGIES FOR TIME DEPENDENT SEISMIC RISK ASSESSMENT

In the time-dependent reliability problem, the hazard rate function  $h(t)$  and the probability of failure  $P_f$ , as define in section 2.3, can be computed once defined the response of a system. In this case, this section treats the methodology to evaluate the response of bridge pier subjected to seismic action and to deterioration process.

#### 6.3.1 SEISMIC ACTION: STOCHASTIC GROUND MOTION MODELS

In this work, the flexible and widely used stochastic point source simulation method of Boore [31] is employed in conjunction with the Atkinson-Silva [32] source-based ground motion model. These allow generating ground motion samples conditional to the features of a given seismic scenario, specified by two main random variables, the moment magnitude  $M$ , and the epicentral distance  $R$ . Thus, the vector  $\mathbf{X}_a$  is defined as  $\mathbf{X}_a = [M, R]$ . The moment magnitude is assumed to follow the Gutenberg-Richter recurrence law [33],  $\nu_M(m) = 10^{(a-bm)}$ , with parameters  $a$  and  $b$  characterising the seismic source (mean number of earthquakes expected) and the regional seismicity (factor governing the proportion of small to large earthquakes), respectively. Given an earthquake event, the aforesaid recurrence law bounded within the range of magnitudes of interest  $[m_0, m_{max}]$  leads to the following probability density function (PDF) of the moment magnitude [33]:

$$f_M(m) = \beta \frac{e^{-\beta(m-m_0)}}{1 - e^{-\beta(m_{max}-m_0)}} \quad (6-34)$$

being  $\beta = b \cdot \log_e(10)$ ,  $m_0$  the magnitude value below which non-significant effects are expected on the structures, and  $m_{max}$  the physical upper bound of the magnitudes expected from the source. The probability density function of  $R$  is obtained under the hypothesis that the source produces random earthquakes with equal likelihood anywhere within a distance from the site  $r_{max}$ , beyond which the seismic effects are assumed to become negligible. The procedure for the simulation of stochastic ground motion time series is summarised in [34].

### 6.3.2 SYSTEM CAPACITY

The seismic capacity of a system is governed by its inherent characteristics, which depend on the properties of its material and geometry. Due to aging and environmental degradation, the system capacity varying continuously with time may be modelled using either stochastic process [35] or deterministic law. In the first case, when the degradation phenomenon is triggered, its evolution occurs at different rates over the time, depending on the mechanism of interest, as shown in [36][37]. In the case of deterministic law, once the propagation rate of the degradation is defined, the process evolves at a constant rate over the time. Additionally, damage can arise from random actions, such as seismic event, producing plastic deformation which may induce loss of capacity. This “shock deterioration” exhibits temporal evolution governed by stochastic process [38]. Over the time, this damage can accumulate depending on the number of occurring and intensity of earthquakes events [39].

In this thesis, the following assumptions are made:

- the state variable influencing the system capacity is the reinforcement area. Environmental degradation of rebars is taken into account by using deterministic laws, expressed as a function of several factors. The phenomenon of chloride corrosion degradation is only considered.
- the capacity is assumed not influenced by the occurrence of seismic events; more in detail, the damage resulting from seismic that do not lead to failure and failure and may occur throughout the structure lifetime is not considered cumulative.

The initiation time of chloride-induced corrosion is evaluated using the Eq. (6-9). As is well known, chloride corrosion induces a reduction in the area of the steel reinforcement, that is the state variable producing different response of the system over the time. This reduction causes a decrease of the element strength. The loss of reinforcement area is evaluated by considering both uniform and pitting corrosion,

following Eq. (6-12) and Eq. (6-14) respectively. However, the effects of corrosion on the strength and ductility of steel rebars, as well as the concrete strength, are not considered.

The vector  $\mathbf{X}_b$  encompasses random variables related to the material properties and the diffusive process. Thus,  $\mathbf{X}_b$  is defined as follows

$$\mathbf{X}_b = [f_c, f_y, \Delta x, D_c, C_{S,\Delta x}, C_{crit}, C_0, i_{corr}, R] \quad (6-35)$$

where  $f_c$  and  $f_y$  are the concrete and the yield strength of steel bar respectively. The pitting factor  $R$  is considered as random variable only in the presence of pitting corrosion. The uncertainties regarding the bar diameter and the concrete cover are neglected.

## 6.4 REFERENCES

- [1] Deodatis, G., and Ellingwood, B.R., and Frangopol, D.M. 2014. Safety, Reliability, Risk and Life-Cycle Performance of Structures and Infrastructures. Boca Raton, FL, USA. CRC Press <https://doi.org/10.1201/b16387>
- [2] Woessner, J., Laurentiu, D., Giardini, D., 2015. The 2013 European Seismic Hazard Model: key components and results. Bull Earthquake Eng 13, 3553–3596. <https://doi.org/10.1007/s10518-015-9795-1>
- [3] DuraCrete. 1998. Modelling of degradation. Duracrete – probabilistic performance based durability design of concrete structures. EU-brite EuRam III. Contract BRPR-CT95-0132. Project BE95-1347/R4-5
- [4] LIFECON 2003 Instructions on methodology and application of models for the prediction of the residual service life for classified environmental loads and types of structures in Europe
- [5] fib. 2006. Fib Bulletin 34: Model code for service life design. Lusanne
- [6] CEN, 2002. EN 1990 Eurocode 0- Basis of Structural Design., Brussels
- [7] ASCE, 2013. Report card for America’s infrastructures. American Society of Civil Engineers.
- [8] Glicksman, Martin E. 2009. Diffusion in Solids: Field Theory, Solid-State Principles, and Application. New York: John Wiley & Sons.
- [9] CEB. 1992. Durable Concrete Structures – Design Guide. Bulletin, 183.
- [10] Biondini, F., Camnasio, E., Palermo, A., 2014. Lifetime seismic performance of concrete bridges exposed to corrosion. Structure and Infrastructure Engineering 10, 880–900. <https://doi.org/10.1080/15732479.2012.761248>
- [11] fib, 2015. Fib Bulletin 76: Benchmarking of deemed to-satisfy provisions in standards. Lusanne
- [12] Chen, D., Mahadevan, S., 2008. Chloride-induced reinforcement corrosion and concrete cracking simulation. Cement and Concrete Composites 30, 227–238. <https://doi.org/10.1016/j.cemconcomp.2006.10.007>
- [13] Val, D.V., Stewart, M.G., Melchers, R.E., 1998. Effect of reinforcement corrosion on reliability of highway bridges. Engineering Structures 20, 1010–1019. [https://doi.org/10.1016/S0141-0296\(97\)00197-1](https://doi.org/10.1016/S0141-0296(97)00197-1)
- [14] González, J.A., Andrade, C., Alonso, C., Feliu, S., 1995. Comparison of rates of general corrosion and maximum pitting penetration on concrete embedded steel reinforcement. Cement and Concrete Research 25, 257–264. [https://doi.org/10.1016/0008-8846\(95\)00006-2](https://doi.org/10.1016/0008-8846(95)00006-2)
- [15] Guo, T., Sause, R., Frangopol, D.M., Li, A., 2011. Time-Dependent Reliability of PSC Box-Girder Bridge Considering Creep, Shrinkage, and Corrosion. J. Bridge Eng. 16, 29–43. [https://doi.org/10.1061/\(ASCE\)BE.1943-5592.0000135](https://doi.org/10.1061/(ASCE)BE.1943-5592.0000135)
- [16] Stewart, M.G., 2009. Mechanical behaviour of pitting corrosion of flexural and shear reinforcement and its effect on structural reliability of corroding RC beams. Structural Safety 31, 19–30. <https://doi.org/10.1016/j.strusafe.2007.12.001>

- [17] Stewart, M.G., Rosowsky, D.V., 1998. Time-dependent reliability of deteriorating reinforced concrete bridge decks. *Structural Safety* 20, 91–109. [https://doi.org/10.1016/S0167-4730\(97\)00021-0](https://doi.org/10.1016/S0167-4730(97)00021-0)
- [18] Rodriguez, J., Ortega, L.M., Casal, J., 1997. Load carrying capacity of concrete structures with corroded reinforcement, *Construction and Building Materials*, vol.11, n.4, 239-248
- [19] Biondini, F., Frangopol, D. (Eds.), 2012. Bridge Maintenance, Safety, Management, Resilience and Sustainability: Proceedings of the Sixth International IABMAS Conference, Stresa, Lake Maggiore, Italy, 8-12 July 2012, Bridge Maintenance, Safety and Management. CRC Press. <https://doi.org/10.1201/b12352>
- [20] Zhang, R., Castel, A., François, R., 2010. Concrete cover cracking with reinforcement corrosion of RC beam during chloride-induced corrosion process. *Cement and Concrete Research* 40, 415–425. <https://doi.org/10.1016/j.cemconres.2009.09.026>
- [21] Coronelli, D., Gambarova, P., 2004. Structural Assessment of Corroded Reinforced Concrete Beams: Modeling Guidelines. *J. Struct. Eng.* 130, 1214–1224. [https://doi.org/10.1061/\(ASCE\)0733-9445\(2004\)130:8\(1214\)](https://doi.org/10.1061/(ASCE)0733-9445(2004)130:8(1214))
- [22] Almusallam, A.A., 2001. Effect of degree of corrosion on the properties of reinforcing steel bars. *Construction and Building Materials* 15, 361–368. [https://doi.org/10.1016/S0950-0618\(01\)00009-5](https://doi.org/10.1016/S0950-0618(01)00009-5)
- [23] Apostolopoulos, C.A., Papadakis, V.G., 2008. Consequences of steel corrosion on the ductility properties of reinforcement bar. *Construction and Building Materials* 22, 2316–2324. <https://doi.org/10.1016/j.conbuildmat.2007.10.006>
- [24] Cairns, J., Plizzari, G.A., Du, Y., Law, D.W., Franzoni, C., 2005 Mechanical Properties of Corrosion-Damaged Reinforcement *MJ* 102. <https://doi.org/10.14359/14619>
- [25] Biondini, F., Camnasio, E., Palermo, A., 2014. Lifetime seismic performance of concrete bridges exposed to corrosion. *Structure and Infrastructure Engineering* 10, 880–900. <https://doi.org/10.1080/15732479.2012.761248>
- [26] Vergani, M., 2010. Modellazione del degrado di strutture in calcestruzzo armato soggette a corrosione. Degree Thesis, Politecnico di Milano, Italy (in Italian).
- [27] Du, Y.G., Clark, L.A., Chan, A.H.C., 2005. Residual capacity of corroded reinforcing bars. *Magazine of Concrete Research*.
- [28] Du Y.: Effect of Reinforcement Corrosion on Structural Concrete Ductility, Ph.D. Thesis, University of Birmingham, UK, Mar 2001, 320 pp.
- [29] Vecchio, F., Collins, M.P., 1986. The modified compression field theory for reinforced concrete elements subjected to shear. *ACI* 83(2), 219-231
- [30] Vidal, T., Castel, A., François, R., 2004. Analyzing crack width to predict corrosion in reinforced concrete. *Cement and Concrete Research* 34, 165–174. [https://doi.org/10.1016/S0008-8846\(03\)00246-1](https://doi.org/10.1016/S0008-8846(03)00246-1)
- [31] Boore, D. M., 2003. Simulation of Ground Motion Using the Stochastic Method. In: *Seismic Motion, Lithospheric Structures, Earthquake and Volcanic Sources*:

- The Keiiti Aki Volume, Birkhäuser Basel, Basel. p. 635–676. doi: 10.1007/978-3-0348-8010-7\_10.
- [32] Atkinson G. M., Silva W., 2000. Stochastic modeling of California ground motions. *Bull Seismol Soc Am* ;90:255–74. <https://doi.org/10.1785/0119990064>.
- [33] Kramer SL., 2003. *Geotechnical Earthquake Engineering*. Englewood Cliffs, NJ.: Prentice-Hall.
- [34] Dall'Asta, A., Scozzese, F., Ragni, L., Tubaldi, E., 2017. “Effect of the damper property variability on the seismic reliability of linear systems equipped with viscous dampers.” *Bulletin of Earthquake Engineering*, 15, 5025-5053. <https://doi.org/10.1007/s10518-017-0169-8>
- [35] Wang, C., 2021. *Structural Reliability and Time-Dependent Reliability*, Springer Series in Reliability Engineering. Springer International Publishing, Cham. <https://doi.org/10.1007/978-3-030-62505-4>
- [36] Li, Q., Wang, C., Ellingwood, B.R., 2015. Time-dependent reliability of aging structures in the presence of non-stationary loads and degradation. *Structural Safety* 52, 132–141. <https://doi.org/10.1016/j.strusafe.2014.10.003>
- [37] Wang, C., Zhang, H., 2020. Assessing the Seismic Resilience of Power Grid Systems Considering the Component Deterioration and Correlation. *ASCE-ASME J Risk and Uncert in Engrg Sys Part B Mech Engrg* 6. <https://doi.org/10.1115/1.4046394>
- [38] Wang, C., Zhang, H., Li, Q., 2017. Reliability assessment of aging structures subjected to gradual and shock deteriorations. *Reliability Engineering & System Safety* 161, 78–86. <https://doi.org/10.1016/j.ress.2017.01.014>
- [39] Biondini, F., Camnasio, E., Titi, A., 2015. Seismic resilience of concrete structures under corrosion. *Earthq Engng Struct Dyn* 44, 2445–2466. <https://doi.org/10.1002/eqe.2591>

## 7 CASE STUDY ANALYSIS

### 7.1 INTRODUCTION

Using the bridge pier from the previous case study as a reference, the chapter examines the time evolution of both its structural capacity and its associated seismic risk. Seismic risk is described by the time-dependent annual rate of exceedance of the ratio between seismic demand and strength capacity. The seismic capacity of the pier is analysed through the bending yield strength of the reinforced cross-section at the base of one its columns. The time evolution of performance is computed in probabilistic terms by considering the random degradation.

Furthermore, a sensitivity analysis is carried out in order to analyse which basic random variables mostly impacts the final probability of collapse in different scenarios.

It is important to highlight that the following results are part of an ongoing research and are still under development. Therefore, they do not fully cover all the topics discussed in the previous section.

### 7.2 DESCRIPTION OF THE CASE STUDY

The influence of degradation on strength is analysed on a reinforced concrete (r.c.) pier of the existing bridge taken as reference in the chapter 5. Each pier has a different height, and it consists of double circular reinforced concrete column with a cross-beam pier-cap with rectangular section (Figure 7-1). The height of the piers ranges between 6.7 to 10.7 m (distance between the top of the basement and the top of the pier-cap cross beam). For the following analysis the pier high 7.7 m is considered. The column cross section has a diameter of 1.40 m and is reinforced with steel bars having nominal diameters  $\phi 18$ , and a concrete cover  $c = 5 \text{ cm}$ . The compressive cylindric strength for the concrete pier is  $f_{ck,op} = 24 \text{ MPa}$ , while the reinforcements are characterized by a characteristic yield strength equal to  $f_{yk,op} = 440 \text{ MPa}$ . Information about bridge geometry and material characteristics has been derived from original drawings and design reports.

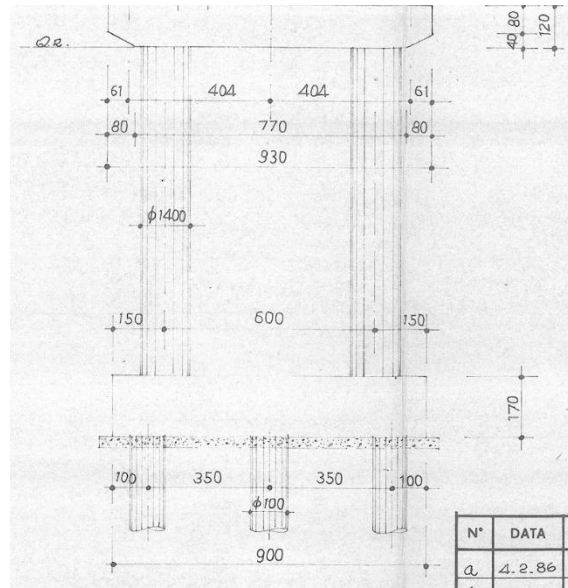


Figure 7-1 Original cross section of bridge pier

### 7.3 TIME EVOLUTION OF CROSS SECTION STATISTICAL PROPERTIES

The time evolution analysis of the statistical properties is carried out observing the yield bending capacity over a lifetime of 100 y. The statistical properties of the basic random variables of capacity are reported in Table 7-1. Their values are based on the following assumptions: exposure class XS1 (since the bridge is near the coast), cement type CEM I and water-cement ratio of 0.45. With these assumptions,  $C_0 = 0$  and  $\Delta x = 0$  are considered, following the recommendation proposed in [1].

The strength of materials, as obtained from the original project, is also assumed as random variable whose probabilities properties are reported in Table 7-2.

Table 7-1 Probability distribution of corrosion random variables

| Corrosion variables | Unit                            | Distribution type                       | $\mu$              | $\sigma$           |
|---------------------|---------------------------------|-----------------------------------------|--------------------|--------------------|
| $D_C$               | $10^{-12} \text{ m}^2/\text{s}$ | Normal                                  | 10<br>[1]          | 2<br>[1]           |
| $C_{S,\Delta x}$    | wt-%/c                          | Log-Normal                              | 4<br>[1]           | 1.50<br>[1]        |
| $C_{crit}$          | wt-%/c                          | Beta( $b_{min} = 0.2$ ; $b_{max} = 2$ ) | 0.2<br>[1]         | 2<br>[1]           |
| $i_{corr}$          | $\mu\text{A}/\text{cm}^2$       | Log-Normal                              | 2.586<br>Table 6-2 | 1.724<br>Table 6-2 |
| $R$                 | /                               | Log-Normal                              | 9.28               | 4.04               |

Table 6-6 Table 6-6

| Table 7-2 Probability distribution of material strength variables |      |                   |                                                |                   |
|-------------------------------------------------------------------|------|-------------------|------------------------------------------------|-------------------|
| Material variables                                                | Unit | Distribution type | $\mu$                                          | $\sigma$          |
| $f_c$                                                             | MPa  | Log-Normal        | $f_{cm} = f_{ck,op} + 8$<br>[2]                | 1.94<br>Table 3-2 |
| $f_y$                                                             | MPa  | Log-Normal        | $f_{ym} = f_{yk,op} + 2 \cdot 30$<br>Table 3-4 | 30<br>Table 3-4   |

Using Eq. (6-9), the deterministic value of  $T_i$  is primarily computed by considering the mean values of basic r.v.. The deterministic value of  $T_i$  is equal to 4 y. Successively, by taking into account the uncertainties of initiation time variables,  $T_i$  becomes a r.v. too with a significant variability. Its behaviour is shown in the histogram in Figure 7-2 in which the vertical line is the deterministic result. The histogram is generated from a MC simulation with a sample size of  $2.0 \cdot 10^6$  realizations. Considering the variable uncertainties, the vales of  $T_i$  exhibit significant dispersion respect the deterministic results. Specially, the most likely values range from 3 to 10 y, although there is a non-negligible probability to exceed 20 y. In this histogram, the  $T_i$  below zero are conventional values and correspond to cases where  $T_i$  exceeds the upper limit, equal to 100 y. It worth nothing that the relatively short initiation time is mainly due to severe condition assumed for exposure, chloride content and diffusion coefficient of concrete.

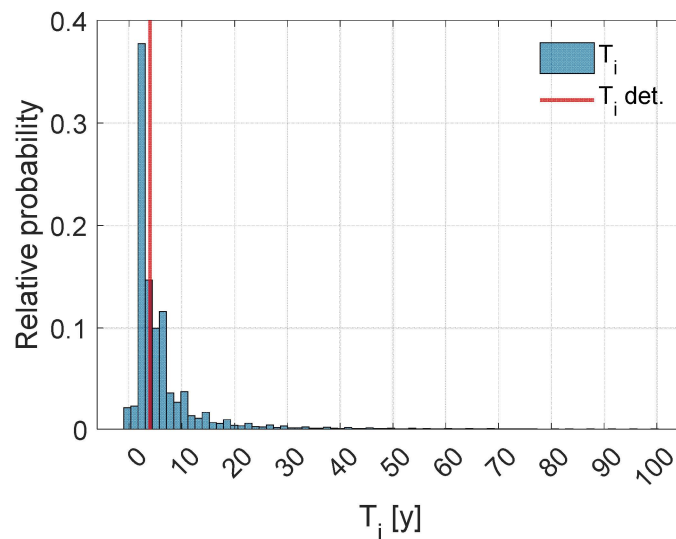


Figure 7-2 Histogram of  $T_i$  with the relative probability to be in each bin

Once  $T_i$  is obtained, the effects of reinforcement corrosion are analysed firstly evaluating the uniform corrosion and then considering pitting corrosion. In both

cases, the deterministic results, adopted as reference, is primarily calculated by considering the mean values of the r.v.. Successively, the uncertainties of r.v. are taken into account to assess their impact on the results.

Considering the uniform corrosion deterministically, the reduction of steel bars is expressed by the ratio of the time varying steel bar to the initial steel bar area  $A_s(t)/A_{s0}$ . The results, illustrated in Figure 7-3 (a), show that the corrosion leads to a loss of 13%, 28%, and 44% of the initial area  $A_{s0}$ , for  $t$  equal to 25, 50, 100 y, respectively. The reduction of the steel bars is reflected in the decrease of the bending moment capacity that is illustrated in Figure 7-3 (b). The undamaged bending capacity  $M_{rd,0}$  is equal to 8574.8 kNm.

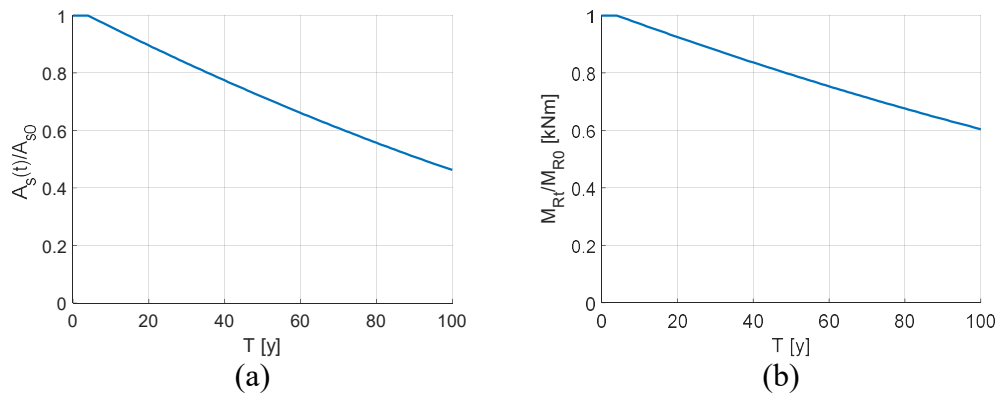


Figure 7-3 (a) deterministic time evolution of the steel bar respect the initial steel bar  $A_s(t)/A_{s0}$ ; (b) deterministic time evolution of the bending capacity respect the undamaged case  $M_{rd}(t)/M_{rd,0}$ ;

By taking into account the uncertainties of corrosion variables,  $A_s(t)$  becomes a r.v. with a large variability due to spread of the initiation time  $T_i$  and the wide dispersion of  $i_{corr}$ . The results, shown in Figure 7-4, are generated from MC simulation with a sample size of  $2.0 \cdot 10^6$  realizations. In the case  $t = 25$  y, only the 7% of the simulations fall within the undamaged category. The remaining cases represent the damaged scenarios, with values that are widely scattered. As the time increases,  $t = 50$  y, the number of undamaged cases decreases, and the corroded values become increasingly dispersed, reaching progressively lower values. This trend becomes particularly evident at  $t = 100$  y, where the results exhibit significant dispersion. At this point, it is possible to observe cases of undamaged, partially corroded with varying degrees of intensity, and fully corroded reinforcing steel.

This behaviour is directly reflected in the estimation of the  $M_{rd}$ , expressed herein by the ratio of the time-varying moment capacity to its undamaged value without considering material uncertainties  $M_{rd}(t)/M_{rd,0}$ . This ratio is reported referring in the case of considered or not the uncertainties of the material strengths, Figure 7-5 (a) and Figure 7-5 (b) respectively.

In Figure 7-5 (a), starting from  $t = 1$  y, when all cases are undamaged, and progressively increasing the observation time when the evolution of the corroded area trend can be observed. In the case of fully corroded of reinforcing steel, the bending capacity dose not reach 0 due to the residual strength provided by concrete. By including the material strength uncertainties Figure 7-5 (b). At time  $t = 1$  y (black bell), the  $M_{rd}$  show a dispersion that is caused only by the variability of the material strength, while the degradation process is not yet triggered. At  $t = 5$  y, although the degradation process is undergoing, its effects are hidden by the variability of material strength. However, as the observation time  $t$  increases, the uncertainties of material become less important than those related to the corrosion.

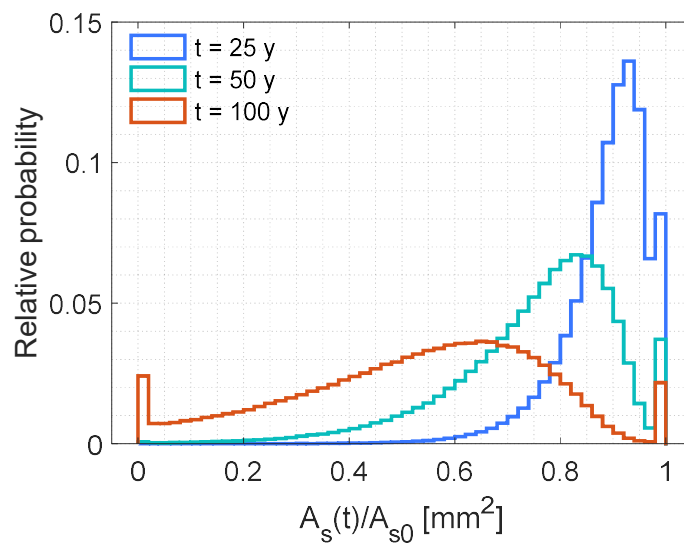


Figure 7-4 Time evolution of the histogram of  $A_s(t)/A_{s0}$  with the relative probability for a time step  $t$  equal to 25 y, 50 y, 100 y

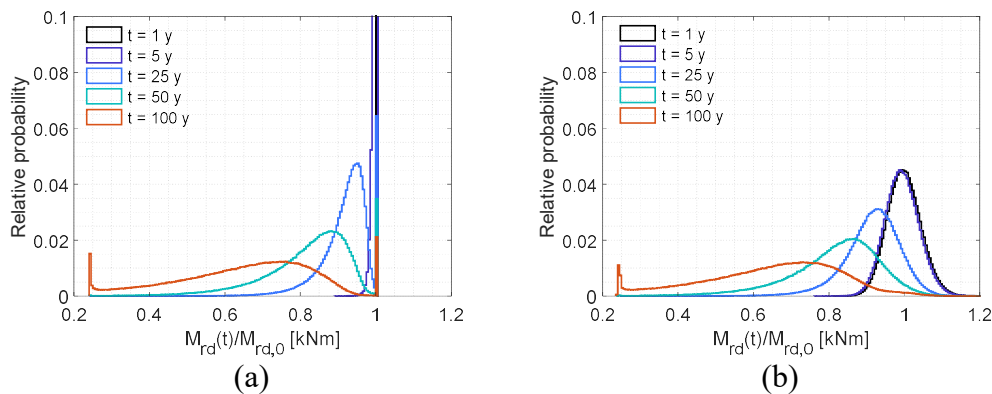


Figure 7-5 (a) histogram of  $M_{rd}/M_{rd,0}$  for several time step  $t$  without strength material uncertainties; (b) histogram of  $M_{rd}/M_{rd,0}$  for several time step  $t$  with strength material uncertainties

Considering pitting corrosion, the penetration ratio  $R$  must be included in vector of the basic random variables, as shown in Table 7-1. Analysed firstly

deterministically, pitting corrosion significantly accelerates the reduction of the steel bar compared to uniform corrosion. The results, illustrated in Figure 7-6 (a), show that pitting leads to complete loss of steel area after 69 y. At this point, the bending moment capacity, Figure 7-6 (b), is reduced to the residual strength provided solely by concrete section.

By taking into account the uncertainties of corrosion variables, Figure 7-7 displays the histograms of  $A_s(t)/A_{s0}$  over the time. In the case of  $t = 25$  y, the histogram shows a large variability compared to the uniform corrosion result. Even though the undamaged cases are greater than the uniform corrosion, there are cases where the reinforcing rebars are fully corroded. This phenomenon is considerably emphasized at  $t = 100$  y where there 40% of simulations are fully corroded, 2% are undamaged and the remaining are partially corroded with several intensities. The penetration ratio  $R$  introduces an additional and no negligible variability in the system response. Moreover, the histogram shape differs from that of uniform corrosion due to pitting corrosion model (Eq. (6-4)) rather than the probability distribution of  $R$ .

This behaviour is directly reflected in the estimation of the  $M_{rd}$ , expressed by  $M_{rd}(t)/M_{rd,0}$  ratio, with and without considering the uncertainties of the material strengths, as shown in Figure 7-8 (a) and Figure 7-8 (b) respectively. The same considerations can be made for the area behaviour in presence of pitting and for the influence of material uncertainties, as shown in Figure 7-5 (b) for uniform corrosion.

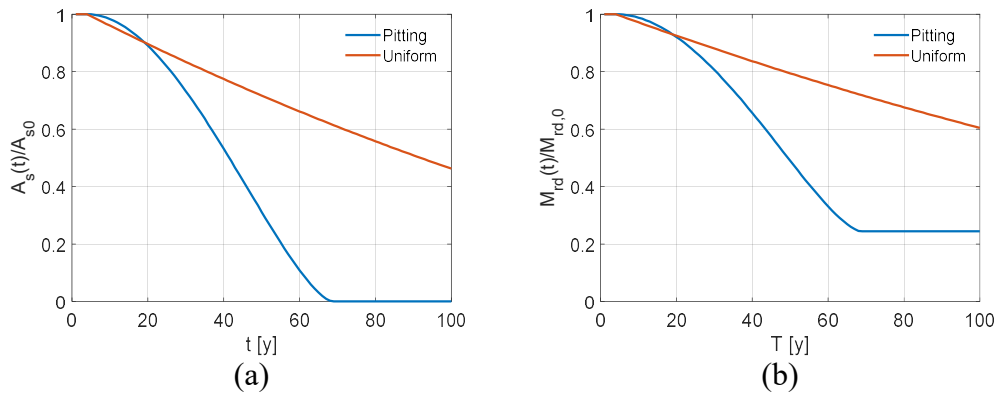


Figure 7-6 Pitting and uniform corrosion: (a) deterministic time evolution of the steel bar respect the initial steel bar  $A_s(t)/A_{s0}$ ; (b) deterministic time evolution of the bending capacity respect the undamaged case  $M_{rd}(t)/M_{rd,0}$ ;

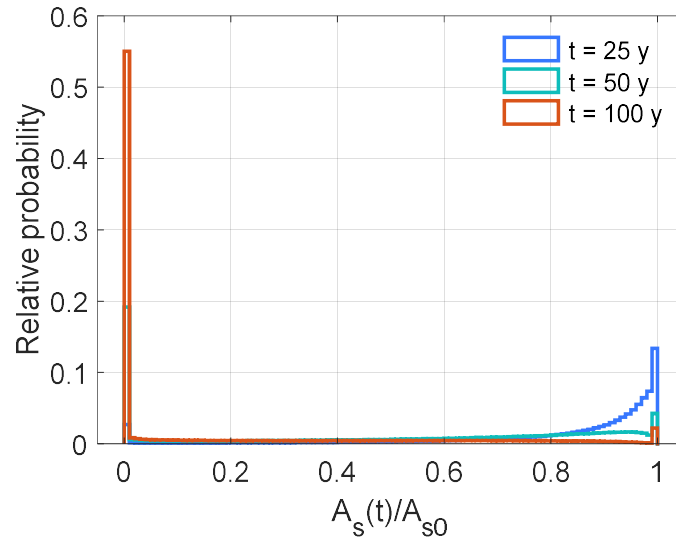


Figure 7-7 Time evolution of the histogram of  $A_s(t)/A_{s0}$  for pitting corrosion with the relative probability for a time step  $t$  equal to 25 y, 50 y, 100 y

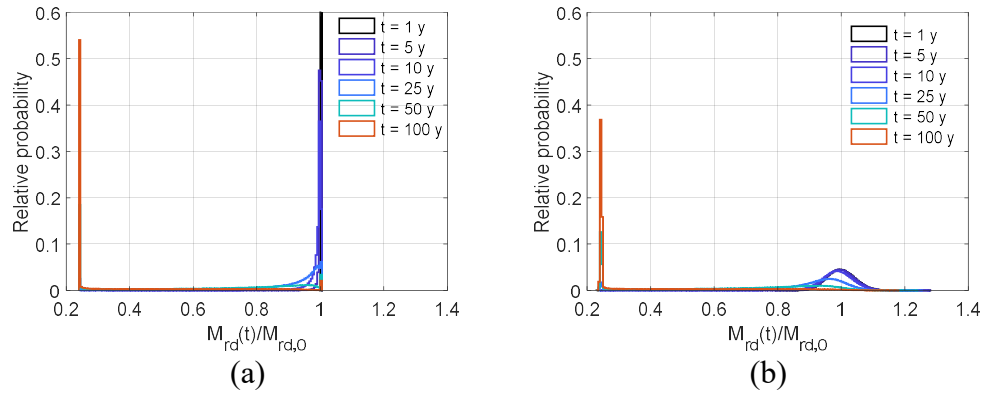


Figure 7-8 Pitting corrosion: (a) histogram of  $M_{rd}/M_{rd,0}$  for several time step  $t$  without strength material uncertainties; (b) histogram of  $M_{rd}/M_{rd,0}$  for several time step  $t$  with strength material uncertainties

## 7.4 SEISMIC HAZARD

A seismic hazard representative of Italian high seismicity zones is adopted in this study. The following set of parameters governing the stochastic hazard model is selected, also according to the existing literature [3]-[5]:  $m_0 = 5.5$ ,  $m_{max} = 8$ ,  $a = 4.35$  and  $b = 0.9$ ,  $r_{max} = 50$  km. A shear wave velocity VS30 equal to 255 m/s has been chosen as representative of deformable soil conditions at the site [4]. The occurrence of the seismic events is described by a Poisson process with a constant hazard rate  $\lambda(t) = \lambda$ .

## 7.5 HAZARD RATES AND FAILURE PROBABILITIES

The system performance is assessed through the response parameter  $e = M_{ed}/M_{rd}$  which expresses the ratio between seismic demand and capacity of the pier in terms of bending moments, evaluated over the time considering only the uniform corrosion. Two limit state threshold can be defined: the value  $e_D = M_{ed}/M_{rd} = 1$  represents the attainment of the Serviceability Limit State (SLS), with the pier approaching its first yielding; the value  $e_U = M_{ed}/M_{rd} = 3$  represents the attainment of the Ultimate Limit State (ULS), and this is in line with the assumption of a behaviour factor  $q = 3$ . In these results only uniform corrosion is considered. In this study, 3 simulation levels ( $i = 3$ ) with  $n_{sim} = 2000$  samples are used, and a set of 5 independent runs of Subset Simulation (Section 2.5.3.2) are performed for each scenario in order to increase the robustness of the estimates. The failure probabilities are estimated considering the average of the 5 simulations.

To assess how different sources of uncertainties affect the risk, various cases of analysis have been considered, with the corresponding hazard rate curves provided in Figure 7-9. Several scenarios have been analysed. In Case 1, the reference condition, the pier is at  $t = t_0$  with no degradation nor additional sources of uncertainty are considered except for the seismic one (black solid line). Case 2 also considers the pier at  $t = t_0$  but incorporates the variability on the structural material properties (green dashed line). Case 3 examines the pier at  $t = t_{100}$  (100 years of service life) in which the uncertainties related to the degradation process are explicitly included (light blue solid line). Case 4 is equal to Case 3 but additionally includes the variability of the structural material properties (red dashed line). Finally, Case 5 also consider the pier at  $t = t_{100}$ , as in Case 3 and includes the degradation process deterministically considering the mean values characterising the degradation related basic random variables from Table 7-1 (yellow solid line). Note that all the analysis scenarios have in common the explicit consideration of the seismic uncertainties.

The hazard rate values  $h(e, t)$  corresponding to the considered limit state ( $e_D, e_U$ ) and referred to all the analysed scenarios are also collected in Table 7-3. The ratios between the hazard rates in each scenario and the reference one  $h(e, t)/h(e, t_0)$  are also reported to quantify the influence on the risk of the different sources of uncertainties.

The analysis of these results (Figure 7-9 and Table 7-3) leads to several considerations. Starting with the SLS, when degradation is included deterministically (Case 5) the exceedance probabilities increase significantly due to the reduced capacity of the system, resulting in a hazard rate 2.5 times higher than the reference case. When the uncertainties associated with the degradation process are included (Case 3), they further amplify the risk, causing a notable upward shift

in the hazard curve and leading to a hazard rate 2.9 times higher than the reference case. These results highlight that both the degradation process and its related uncertainties must be considered in the structural assessment over the time of a bridge pier subjected to a seismic risk. Moreover, the influence of material uncertainties, as observed in Case 2 and Case 4, is negligible, as expected.

For the ULS, the differences among the cases are more pronounced than in the SLS. While in the SLS the hazard curves for deterministic corrosion (Case 5) and uncertain corrosion (Case 3) are relatively close, in the ULS, they diverge significantly leading to hazard rate values differing by an order of magnitude. This behaviour arises from the combined effect of greater variability of the seismic action at ULS, where extreme events introduce a higher degree of uncertainty of effect of action, and the deterioration uncertainties, that amplifies the uncertainty of structural capacity. As a results, the hazard rate increases significantly. This result highlights the necessity to include corrosion uncertainties in seismic risk assessment. The effect of material uncertainties (Case 2 and Case 4) produces a more pronounced variation in the hazard rate ratio compared to the SLS; however, their impact remains marginal respect that given by the degradation process.

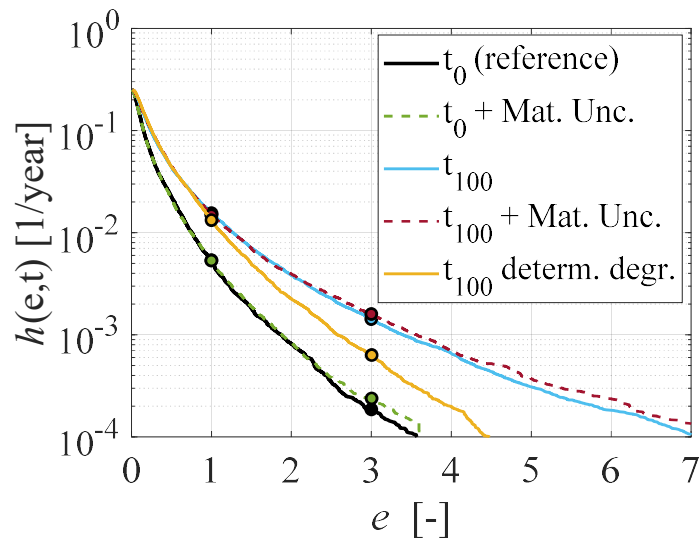


Figure 7-9 Demand hazard curves of the ratio between the seismic demand and pier capacity for different analysis scenario referring to serviceability conditions ( $e_D = 1$ ) and ultimate conditions ( $e_U = 3$ )

Table 7-3 Failure probabilities for different cases of analysis and related to different LS

| Case of analysis | $h(e_D, t)$ | $h(e_D, t) / h(e_D, t_0)$ | $h(e_U, t)$ | $h(e_U, t) / h(e_U, t_0)$ |
|------------------|-------------|---------------------------|-------------|---------------------------|
|                  | [1/years]   | [-]                       | [1/years]   | [-]                       |

|                            |        |      |        |      |
|----------------------------|--------|------|--------|------|
| $t_0$ (reference)          | 0.0054 | 1.00 | 0.0002 | 1.00 |
| $t_0$ +Mat. Unc.           | 0.0053 | 1.00 | 0.0002 | 1.28 |
| $t_{100}$                  | 0.0155 | 2.89 | 0.0014 | 7.67 |
| $t_{100}$ + Mat. Unc.      | 0.0149 | 2.77 | 0.0016 | 8.59 |
| $t_{100}$ determ.<br>degr. | 0.0133 | 2.47 | 0.0006 | 3.40 |

In order to assess how the failure probability change over the lifetime (and also quantify the influence of a time-dependent assessment of the failure probability) and considering the corrosion uncertainties, a set of probabilistic analyses have been performed at different instants of time: 0, 25, 50, 75, 100 y. The corresponding hazard curves are shown in *Figure 7-10 (a)*, while the chart of *Figure 7-10 (b)* provides a graphical representation of the hazard rates variation over the considered time intervals. In *Figure 7-10 (b)*, the blue lines refer to SLS, while the red ones to the ULS; moreover, the dashed lines identify the time-invariant condition in which the hazard rate is constant (no corrosion) and equal to that estimate at  $t = t_0$ . These results are also collected in Table 7-4 including an explicit calculation of the corresponding failure probabilities. The ratios between the time-dependent estimation of failure probabilities and the time-invariant  $P_f/P_{f,0}$  one is also reported.

These results strengthen the previous observations. As expected, introducing the deterioration with its uncertainties leads to higher the failure probability  $P_f$  at the same time  $t$  compared to the case with constant hazard rate. For the SLS, neglecting deterioration while increasing  $t$  results in an underestimation of  $P_f$  by up to 39% for  $t = 100$  y. This divergence becomes more pronounced in the case of ULS as it can be observed for the results of  $P_f/P_{f,0}$  in Table 7-4.

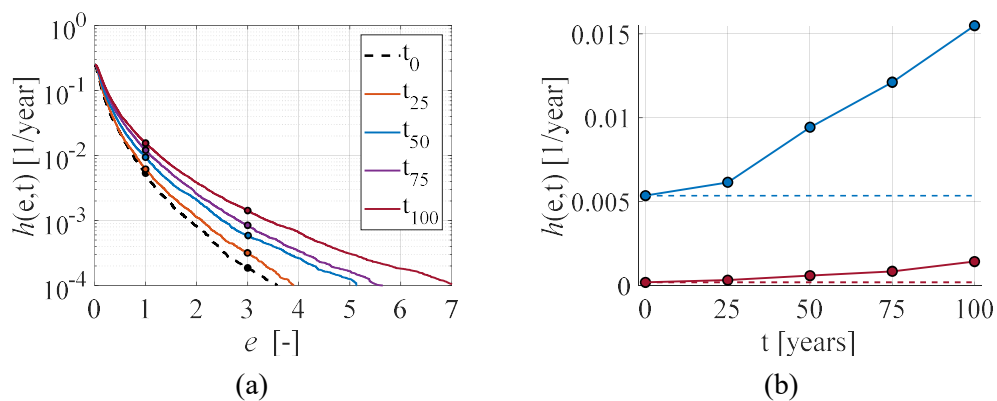


Figure 7-10 (a) hazard rate curves for different instant of time; (b) graphical representation of hazard rate for SLS (blue line) and ULS (red line)

*Table 7-4 Failure probabilities for Serviceability and Ultimate Limit State at different time within the lifetime*

| $t$<br>[y] | <i>SLS</i>            |                          |                      |                      | <i>ULS</i>            |                          |                      |                      |
|------------|-----------------------|--------------------------|----------------------|----------------------|-----------------------|--------------------------|----------------------|----------------------|
|            | $h(e_D, t)$<br>[1/yr] | $P_{f,0}(e_D, t)$<br>[%] | $P_f(e_D, t)$<br>[%] | $P_f/P_{f,0}$<br>[-] | $h(e_U, t)$<br>[1/yr] | $P_{f,0}(e_U, t)$<br>[%] | $P_f(e_U, t)$<br>[%] | $P_f/P_{f,0}$<br>[-] |
| 0          | 0.0054                | //                       | //                   | //                   | 0.0002                | //                       | //                   | //                   |
| 25         | 0.0062                | 12.5                     | 13.4                 | 1.1                  | 0.0003                | 0.5                      | 0.6                  | 1.4                  |
| 50         | 0.0094                | 23.5                     | 28.8                 | 1.2                  | 0.0006                | 0.9                      | 1.8                  | 1.4                  |
| 75         | 0.0121                | 33.1                     | 45.6                 | 1.3                  | 0.0008                | 1.4                      | 3.5                  | 1.5                  |
| 100        | 0.0155                | 41.5                     | 61.5                 | 1.4                  | 0.0014                | 1.9                      | 6.2                  | 2.8                  |

## 7.6 SENSITIVITY OF RANDOM PARAMETERS

Finally, thanks to the particular features of the algorithm, Subset Simulation allows to understand how the basic variables distribution changes moving from frequent to rare events, and this make possible to highlight the most influencing basic RVs involved into the problem. Looking at the Corner Plot of Figure 7-11 it is possible to see that, as expected, magnitude  $M$  and epicentral distance  $R$  strongly change on the diagonal plots, i.e. the marginal distribution strongly changes increasing the subset level. On the degradation process side instead, among the many basic variables considered only  $C_{crit}$  and  $i_{corr}$  change, with the first clearly moving to lower values on the marginal distribution. This happens because, lower values of  $C_{crit}$  produce shorter corrosion initiation times, while higher  $i_{corr}$  values accelerate the corrosion process. The joint distributions of any pair of variables confirms that there isn't any significant relation between the basic variables, as expected.

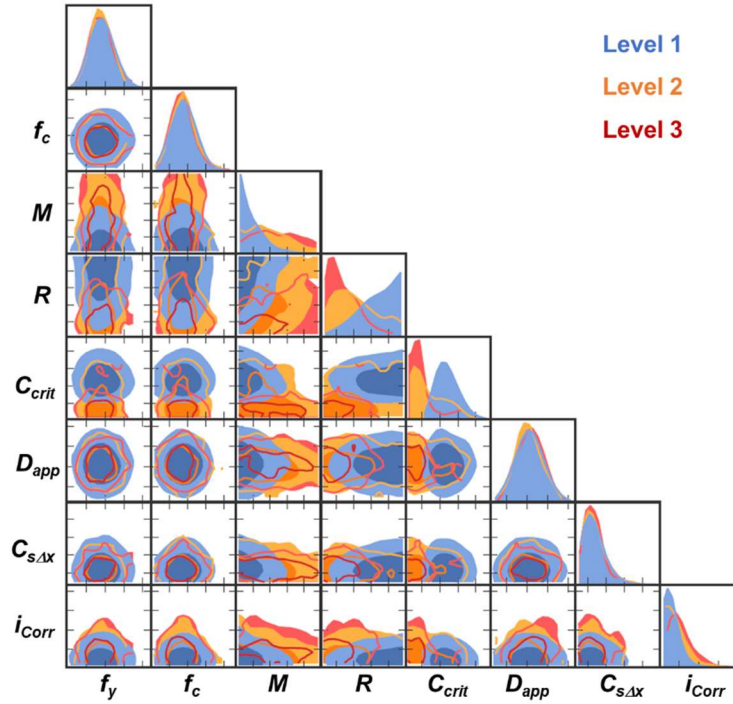


Figure 7-11 Corner Plot of the basic RVs for different levels of simulation

In Figure 7-12, the corner plot is presented to assess the correlation between engineering response parameters and degradation basic RVs. As expected, the seismic demand  $M_{ed}$  is strongly influenced by the PGA, function of Distance and Magnitude. On degradation process side, the initiation time  $T_i$  is primarily influenced by the  $C_{crit}$  and marginally affected by other parameters characterizing the diffusion process.  $T_i$  produces a minor effect on reinforcement area  $A_s$ , which is more significantly influenced by the  $i_{corr}$ . This is because, even though  $T_i$  reaches lower value on its marginal distribution, the possible values of  $A_s$  over the time are strictly correlated with the  $i_{corr}$  that is the only variable governing the evolution of degradation.

Furthermore, after the first run of Subset analysis (standard Monte Carlo simulation), the marginal distribution of  $A_s$  exhibits a binomial distribution due to the effect of deterioration process variables. As expected,  $M_{rd}$  is directly influenced by the reinforcement area  $A_s$ , as observed by the diagonal plot, and also follows a binomial distribution.

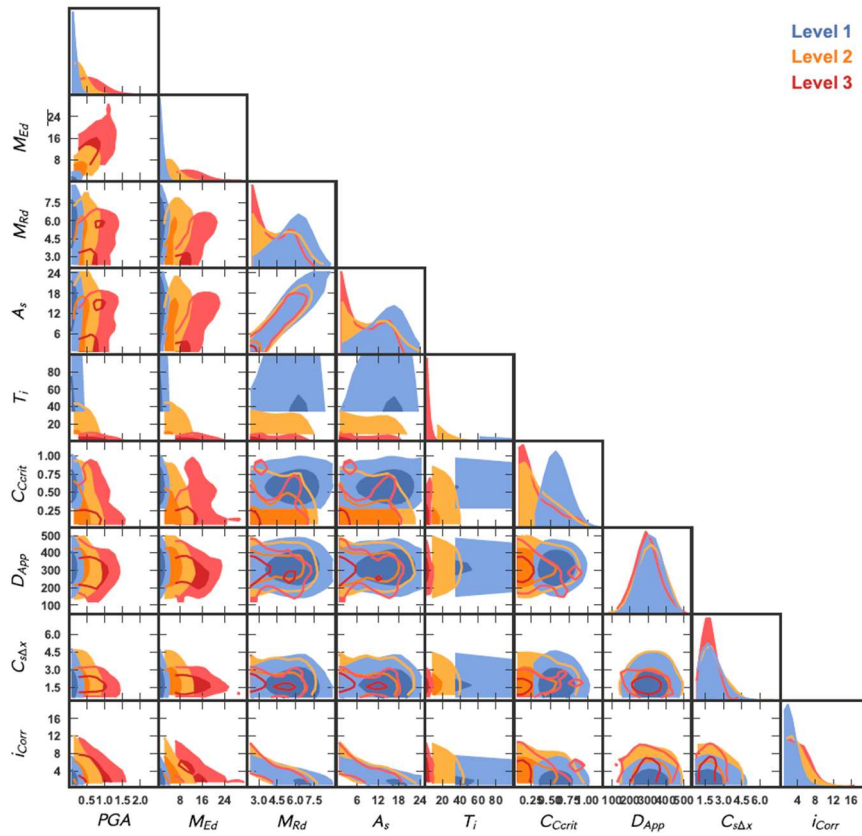


Figure 7-12 Corner Plot of both engineering response parameters and degradation basic RVs, for different levels of simulation.

## 7.7 REFERENCES

- [1]fib, 2015. Fib Bullettin 76: Benchmarking of deemed to-satisfy provisions in standards. Lusanne
- [2]CEN, 2002. EN 1990 Eurocode 0- Basis of Structural Design., Brussels
- [3]Taflanidis A. A., Beck J. L., 2018. Life-cycle cost optimal design of passive dissipative devices. *Struct Saf* 2009;31(6):508–22. [48] Altieri D, Tubaldi E, De Angelis M, Patelli E, Dall'Asta A. Reliability-based optimal design of nonlinear viscous dampers for the seismic protection of structural systems. *Bull Earthq Eng*;16(2):963–82
- [4]Boore DM, Joyner WB., 1997 Site amplifications for generic rock sites. accessed 22 January *Bull Seismol Soc Am*; 87:327–41. <http://www.bssaonline.org/content/87/2/327.short>.
- [5]Micozzi, F., Scozzese, F., Ragni, L., Dall'Asta, A., 2022. Seismic reliability of base isolated systems: sensitivity to design choices. *Engineering Structures*, 256, 114056

## 8 CONCLUSION

This study was conducted to provide a comprehensive reliability assessment of existing post-tensioned concrete bridges. Specifically, the research aims to evaluate the impact of knowledge uncertainties on the reliability assessment of post-tensioned (PT) beams under vertical loads and to examine the combined effects of seismic events and structural degradation due to environmental factors.

The first objective was pursued by analysing the effects of knowledge uncertainties using both semi-probabilistic and full-probabilistic approaches on an existing PT beam belonging to a bridge case study. The knowledge process is simulated by the Monte Carlo method, where the key parameters that play a crucial role are *i*) the number  $n$  of tests, *ii*) the confidence level  $\alpha$  used for the estimation of the design parameters, and *iii*) the coefficient of variation  $c_{v_e}$  of the measurement error. The results revealed both strengths and weakness of the safety format applied to this type of structures. The safety format is robust since different levels of  $n$  and  $\alpha$  lead almost always to a conservative estimation of the capacity/demand ratio. As expected, the probability to have a safety estimation of the safety ratio increases by increasing the number of tests and the confidence level. For the value of confidence  $\alpha$  equal to 0.75, recommended by the Eurocodes the probability  $F_{\bar{Y}_d}(y_d)$  to have a safety estimation of  $\bar{Y}_d$  is high in both the scenarios, with and without the measurement error. Results depend on the expressions used for the estimation of the design parameters  $\bar{\Theta}_d$ , and relevant underlying assumption. The Eurocode formulas, based on Coverage Method and t-Student method, provide satisfactory results. Moreover, it was also observed that high values of the confidence factor  $\alpha$ , approximately larger than 0.8, do not notably improve the result  $F_{\bar{Y}_d}(y_d)$ , but significantly reduce the expected value  $\bar{Y}_{d,m}$  of  $\bar{Y}_d$ . Thus, it becomes very probable to obtain under-estimation of the safety ratio, and, in a misleading way, the structure could appear to be not adequate. Furthermore, measurement error substantial impact on the results, increasing the probability of underestimating the structural capacity and providing results more and more conservative.

By extending the analysis to a fully probabilistic approach, where failure probability is directly estimated, the impact of knowledge uncertainties became even more pronounced. In the scenario that neglects the measurement error, adopting a confidence level  $\alpha$  equal to 0.75 provide a good balance between reliability and economic efficiency. However, when the measurement error is included, even at this  $\alpha$ , the results diverge significantly from the reference case. This difference is particularly notable in the simulation method, where all knowledge uncertainties are incorporated into the estimated distribution of the basic random variables. These results, referring to PT bridges girder typology, highlight

the requirements of further investigations, especially regarding more precise information about measurement error relevant to specific techniques used for the knowledge of the prestressing system.

The uncertainties affecting the reliability of bridges are further amplified when considering the combined effect of seismic event and structural deterioration, which formed the second objective of this study. The results demonstrated that degradation uncertainties significantly affect the structural seismic capacity. Neglecting these uncertainties results in a significant overestimation of structural capacity. By analysing their influence on pier strength of the case study, considerable dispersion emerges in the potential values of the remaining reinforcing steel and, consequently, in the bending strength. This variability becomes increasingly pronounced over time. Furthermore, including seismic events, it is evident that the uncertainties related to corrosion variables cannot be neglected. These uncertainties significantly increase the annual hazard rate compared to scenario where degradation is considered deterministically. This difference becomes very important when Ultimate Limit State conditions are examined, emphasizing the importance of including these uncertainties in assessments. These results become more evident assessing how the failure probability change over the lifetime of bridges. As expected, neglecting the deterioration and its uncertainties leads to an underestimation of the failure probability that increase over the time. This effect is even more pronounced in the Ultimate Limit State. The uncertainties related to mechanical properties have negligible influence. However, not all basic random variables contribute equally to the system's response. Alongside seismic parameters such as magnitude and epicentral distance, critical chloride concentration and chloride-induced corrosion rate emerged as the primary factor influencing structural behaviours, whereas other variables have minimal impact. These results are useful with the aim to consider an essential number of basic random variables. These results are part of an ongoing research project and therefore are preliminary. Future developments could focus on evaluating: the life-cycle performance of bridge piers, incorporating the combined effects of seismic damage and the reductions in strength and ductility caused by corrosion. Moreover, by considering the seismic isolation both for its use in design of new bridge and for its application of retrofit intervention in existing ones, it is significant to interest to focus on time dependent reliability of isolated bridges. This involves incorporating additional sources of uncertainties of the system response that, beyond the mechanical properties of the devices, include those of their aging, deterioration and malfunctions. All these aspects play a main role in the seismic response of isolated bridges.

# A. APPENDIX: STATISTICAL CONCEPTS

## RANDOM VARIABLES

A random variable [1] is a mathematical vehicle for representing an event in analytical form. Contrary to a deterministic variable, which can assume a definite value, a random variable may be defined within a range of possible values. A random variable  $X$  may be described as a mapping function that convert the events in a sample space  $\Omega$  into the real numbers. Random variables are discrete, if they assume a finite (or countable) set of values, or are continuous, if their values are uncountable. Hereafter only the continuous variables will be treated since they are more widely used in structural engineering.

The numerical values, that a random variable can be assumed, are associated with a probability of measures. Let  $X$  be a random variable with a probability function  $\mathbb{P}$ , the cumulative distribution function (CDF) of  $X$ ,  $F_X(x)$  is denoted as follows

$$F_X(x) = \mathbb{P}(X \leq x), \text{ for } \forall x \in (-\infty, \infty) \quad (\text{A-1})$$

In addition to CDF, the random variable  $X$  is described by probability density function (PDF),  $f_X(x)$  for  $x \in (-\infty, \infty)$ , defined by

$$\int_{\mathcal{D}} f_X(x) dx = \mathbb{P}(X \in \mathcal{D}) \quad (\text{A-2})$$

Where  $\mathcal{D}$  is a set of real numbers,  $\mathcal{D} \subset (-\infty, \infty)$ . The relationship between CDF and PDF (Figure A-1) of a continuous variable  $X$  is determined by considering Eq. (A-1), which gives

$$F_X(x) = \mathbb{P}(X \leq x) = \mathbb{P}(-\infty \leq X \leq x) = \int_{-\infty}^x f_X(\tau) d\tau \quad (\text{A-3})$$

Aking the differential from of both sides of Eq. (A-3) further yields

$$f_X(x) = \frac{dF_X(x)}{dx} \quad (\text{A-4})$$

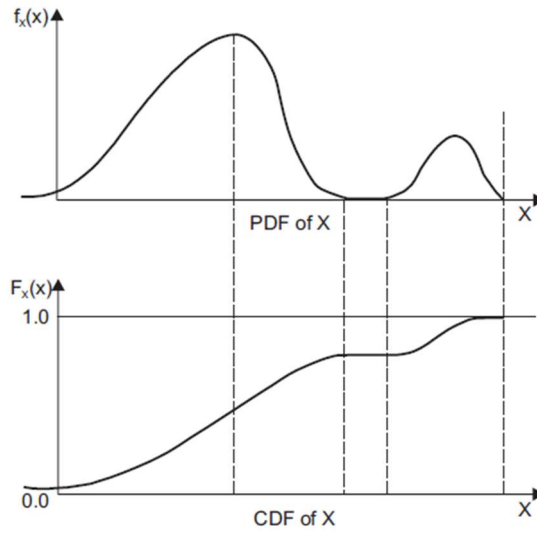


Figure A-1 PDF and CDF of continuous random variable [1]

The distribution functions of a random variable are completely and uniquely described if some parameters (main descriptors) are specified. The number of parameters needed is function of the nature of the uncertainty; in most cases these parameters can be estimated from the information of central values, dispersion, skewness, kurtosis.

A random variable can assume multiple values, each of which with an associated probability within a given range. Determining a central value within this range is particularly important. Specially, the ‘weighted average’, also known as the mean value or expected value, denoted as  $E(X)$ , that for a continuous random variable  $X$  with PDF  $f_X(x)$  is given by

$$E(X) = \mu_X = \int_{-\infty}^{\infty} x f_X(x) dx \quad (A-5)$$

Other quantities used to indicate the central value of a random variable are the mode and the median. The mode is the most probable value of a random variable, while the median is the value at which the CDF is 50%.

A measure of dispersion is needed to indicate how widely or narrowly the values of random variable are dispersed. If the deviations are defined with respect to the mean value, the measure of dispersion is the variance, denoted as  $Var(X)$  or  $\sigma_X^2$ , that for a continuous random variable  $X$  with PDF  $f_X(x)$  is

$$Var(X) = \sigma_X^2 = \int_{-\infty}^{\infty} (x - \mu_X)^2 f_X(x) dx \quad (A-6)$$

A suitable measure of dispersion is the standard deviation, that is

$$\sigma_X = \sqrt{\text{Var}(X)} \quad (\text{A-7})$$

If  $\mu(x) > 0$ , it is often preferred a nondimensional measure of variability that is the coefficient of variation  $c_v$

$$c_v = \frac{\sigma_X}{\mu_X} \quad (\text{A-8})$$

The skewness gives a measure of symmetry or asymmetry of the PDF and it is defined as follows

$$E(X - \mu_X)^3 = \int_{-\infty}^{\infty} (x - \mu_X)^3 f_X(x) dx \quad (\text{A-9})$$

The skewness can be 0 if PDF is symmetric with respect to the mean value, positive if the values of  $X$  above  $\mu_X$  are more widely dispersed than the dispersion of the values below  $\mu_X$ , negative otherwise. The dimensionless measure of skewness is the skewness coefficient

$$\theta = \frac{E(X - \mu_X)^3}{\sigma_X^3} \quad (\text{A-10})$$

Finally, a useful property is the kurtosis that is a measure of the peakedness of the PDF and it is defined as follow

$$E(X - \mu_X)^4 = \mu_4 = \int_{-\infty}^{\infty} (x - \mu_X)^4 f_X(x) dx \quad (\text{A-11})$$

## JOINTLY DISTRIBUTED RANDOM VARIABLES

Some events are the results of two or more physical processes. If these processes can be modelled as random variables defined on the same probability space, their possible combinations, and hence the possible values of their corresponding event, may be analysed using the joint probability distribution. For the sake of simplicity, the discussion of the joint probability distribution is showed for the case of two random variables. The extension of the discussion to more random variables will be obvious.

Let  $X$  and  $Y$  two random variables, the probability of their different combinations of outcome is described with the joint distribution probability, whose CDF is

$$F_{X,Y}(x, y) = P(X \leq x, Y \leq y) \quad (A-12)$$

If the random variables are continuous, the joint probability distribution may be described with the joint PDF as follows

$$f_{X,Y}(x, y) = \frac{\partial^2 F_{X,Y}(x, y)}{\partial x \partial y} \quad (A-13)$$

The values of the two random variables may be statistically dependent with each other; in this case it is useful to calculate the conditional probability density function. Let  $X$  and  $Y$  be continuous random variables, the conditional PDF of  $X$  given  $Y$  is

$$f_{X|Y}(x|y) = \frac{f_{X,Y}(x, y)}{f_Y(y)} \quad (A-14)$$

If  $X$  and  $Y$  are statistically independent  $f_{X|Y}(x|y) = f_X(x)$ , the joint PDF becomes

$$f_{X,Y}(x, y) = f_X(x)f_Y(y) \quad (A-15)$$

In some cases, starting from the joint PDF of  $X$  and  $Y$  and by completely removing the effect of  $Y$ , the marginal PDF of  $X$  is obtained as follow

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x, y) dy \quad (A-16)$$

The joint PDF of two random variables and the related marginal PDF are shown in Figure A-2.

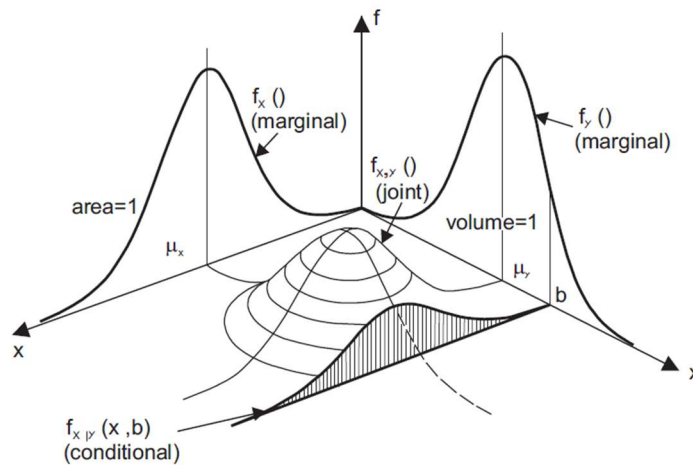


Figure A-2 Joint, conditional and marginal PDF

## PROBABILISTIC DISTRIBUTIONS USED

This section briefly introduced the most commonly used probability distribution functions in structural reliability analysis. For detailed information on all the distribution functions refer to [1]-[3].

### NORMAL OR GAUSSIAN DISTRIBUTION

The Normal or Gaussian distribution is one of the most widely used distribution. The PDF for a random variable  $X$  can be expressed as

$$f_X(x) = \frac{1}{\sigma_X \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{x - \mu_X}{\sigma_X} \right)^2 \right], -\infty < x < \infty \quad (\text{A-17})$$

where  $\mu_X$ ,  $\sigma_X$  are the mean and the standard deviation respectively. The corresponding CDF can be expressed as

$$F_X(x) = \int_{-\infty}^x \frac{1}{\sigma_X \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{x - \mu_X}{\sigma_X} \right)^2 \right] dx \quad (\text{A-18})$$

This distribution, denoted as  $N(\mu, \sigma)$ , is symmetric about the mean and the mean, the median and the modal values coincide.

The normal distribution of a random variable  $X$  can be standardized into a distribution having zero mean and unit standard deviation as

$$S = \frac{X - \mu_X}{\sigma_X} \quad (\text{A-19})$$

The PDF and CDF of the standard normal distribution are expressed in Eq. (A-20) and Eq. (A-21) respectively.

$$f_S(s) = \frac{1}{\sqrt{2\pi}} \exp \left[ -\frac{1}{2} s^2 \right], -\infty < s < \infty \quad (\text{A-20})$$

$$F_S(s) = \int_{-\infty}^s \frac{1}{\sqrt{2\pi}} \exp \left[ -\frac{1}{2} s^2 \right] ds \quad (\text{A-21})$$

### LOGNORMAL DISTRIBUTION

The lognormal distribution is also a common distribution that allows to eliminate the possibility of negative values. If a random variable has a lognormal

distribution, then its natural logarithm has a normal distribution. For a random variable  $X$  the PDF is

$$f_X(x) = \frac{1}{\zeta_X \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{\ln x - \lambda_X}{\zeta_X} \right)^2 \right], 0 < x < \infty \quad (\text{A-22})$$

where  $\lambda_X$  and  $\zeta_X$  are respectively the mean and standard deviation of  $\ln X$ . These two parameters can be calculated, using the Eq. (A-23) and Eq. (A-24), from the information on the two parameters of the normal distribution. It can be showed that

$$\lambda_X = \ln \mu_X - \frac{1}{2} \zeta_X^2 \quad (\text{A-23})$$

$$\zeta_X^2 = \ln \left( 1 + \left( \frac{\sigma_X}{\mu_X} \right)^2 \right) \quad (\text{A-24})$$

Analogously to the normal distribution, the lognormal one can be standardized too as

$$S = \frac{\ln X - \lambda_X}{\zeta_X} \quad (\text{A-25})$$

#### POISSON AND EXPONENTIAL DISTRIBUTION

Given a time period  $[0, T]$ , if events may randomly occur at any instant, the occurrence of the event may be modelled as a Poisson process. Dividing the interval into  $n$  identical sections, having each a duration of  $\Delta t$ , the Poisson process is based on the following assumptions:

- An event can randomly occur at any time instant
- The occurrence of an event in a given time interval is statistically independent of those occurring in other intervals
- The probability of occurrence  $p$  of an event in a small interval  $\Delta t$  is proportional to  $\Delta t$  and can be given by  $\lambda \Delta t$ , where  $\lambda$  is the mean occurrence rate of the event (assumed constant)
- The probability of two or more occurrences in  $\Delta t$  is negligible (in higher order of  $\Delta t$ )

Based on these assumptions, let  $X$  a Poisson random variable representing the number of events in the time period  $[0, T]$ , the probability of  $X = k$ , where  $k$  is an arbitrary non-negative integer, can be evaluated by

$$P(X = k) = \frac{(\lambda T)^k}{k!} e^{-\lambda T}, k = 1, 2, \dots \quad (\text{A-26})$$

The mean value and variance of  $X$  are both equal to  $\lambda T$ .

If the occurrence of an event constitutes a Poisson process, the recurrence time would be described by an exponential distribution. The probability of a first occurrence of an event at the time instant  $T_1 > t$ , assuming no occurrence in the range  $(0, t)$ , can be described by Eq. (A-27)

$$P(T > t) = e^{-\lambda t} \quad (\text{A-27})$$

where  $\lambda$  is the occurrence rate.  $T_1$  can be even the time between two consecutive occurrences of the same event, since the occurrences of an event in nonoverlapping intervals are statistically independent.

The CDF and PDF of exponential distribution of  $T$  are defined in Eq. (A-28) and Eq. (A-29) respectively.

$$F_T(t) = 1 - e^{-\lambda t} \quad (\text{A-28})$$

$$f_T(t) = \lambda e^{-\lambda t} \quad (\text{A-29})$$

The mean value  $\mu_X$ , denoted also return period  $T_R$ , and variance  $\sigma_X^2$  of occurrence time are defined in Eq. (A-30) and Eq. (A-31) respectively.

$$\mu_X = T_R = \frac{1}{\lambda} \quad (\text{A-30})$$

$$\sigma_X^2 = \frac{1}{\lambda^2} \quad (\text{A-31})$$

## BETA DISTRIBUTION

Beta distribution is used for random variable whose possible values are bounded by two finite limits,  $a$  and  $b$ . The PDF of this distribution is given by

$$f_X(x) = \frac{1}{B(q, r)} \frac{(x-a)^{q-1}(b-x)^{r-1}}{(b-a)^{q+r-1}}, a \leq x \leq b \quad (\text{A-32})$$

*, elsewhere*

where  $q$  and  $r$  are the parameters of the distribution and  $B(q, r)$  is the beta function. The distribution parameters can be derived from the mean  $\mu_X$  and the standard deviation  $\sigma_X$  of the available data as follows

$$\mu_X = a + \frac{q}{q+r}(b-a) \quad (\text{A-33})$$

$$\sigma_X^2 = \frac{qr}{(q+r)^2(q+r+1)}(b-a)^2 \quad (\text{A-34})$$

$B(q, r)$  can be calculated as

$$B(q, r) = \frac{\Gamma(q)\Gamma(r)}{\Gamma(q+r)} \quad (\text{A-35})$$

where  $\Gamma()$  is the gamma function. If the lower limit  $a$  is 0 and the upper limit is 1, the distribution becomes standard, and its PDF takes the following form

$$f_X(x) = \begin{cases} \frac{1}{B(q, r)}(x)^{q-1}(1-x)^{r-1}, & 0 \leq x \leq 1 \\ 0, & \text{elsewhere} \end{cases} \quad (\text{A-36})$$

When  $q$  and  $r$  are equal the distribution becomes a uniform distribution.

#### GUMBEL DISTRIBUTION

The CDF of the Gumbel distribution of the largest value  $Y_n$  of a random variable  $X$ , also called Type I Extreme Value Distribution, can be expressed as

$$F_{Y_n}(y_n) = \exp[-e^{a_n(y_n-u_n)}] \quad (\text{A-37})$$

where  $u_n$  is the characteristic largest value of the initial variable  $X$ , and  $a_n$  is the inverse measure of dispersion of the largest value of  $X$ . The corresponding PDF is

$$f_{Y_n}(y_n) = a_n e^{-a_n(y_n-u_n)} \exp[-e^{-a_n(y_n-u_n)}], -\infty < y_n < \infty \quad (\text{A-38})$$

The parameters  $u_n$  and  $a_n$  can be derived from the mean and standard deviation of the extreme value variable  $Y_n$  as

$$a_n = \frac{1}{\sqrt{6}} \left( \frac{\pi}{\sigma_{Y_n}} \right) \quad (\text{A-39})$$

$$u_n = \mu_{Y_n} - \frac{0.5772}{a_n} \quad (\text{A-40})$$

The distribution for the smallest value can be found in the literature.

## WEIBULL DISTRIBUTION

Let  $X$  be a random variable, the PDF of the Weibull distribution is expressed as follows

$$f_X(x; \lambda, k) = \begin{cases} \frac{k}{\lambda} \left(\frac{x}{\lambda}\right)^{k-1} e^{-(x/\lambda)^k}, & x \geq 0 \\ 0, & x < 0 \end{cases} \quad (\text{A-41})$$

where  $k$  is the shape parameter and  $\lambda$  is the scale parameter. The mean and the standard deviation of this distribution can be expressed as

$$\mu_X = \lambda \Gamma\left(1 + \frac{1}{k}\right) \quad (\text{A-42})$$

$$\sigma_X^2 = \lambda^2 \left[ \Gamma\left(1 + \frac{2}{k}\right) + \left(\Gamma\left(1 + \frac{1}{k}\right)\right)^2 \right] \quad (\text{A-43})$$

where  $\Gamma()$  is the gamma function.

## STATISITICAL INFERENCE

The previous section discusses the probability distributions of a random variable and the estimation of its parameters, which are usually derived from real-world data. For example, determining the characteristic strength of a structure material relies on information obtained from in-situ tests. Similarly, estimating the maximum number of heavy vehicles on a highway bridge, repeated counts of vehicles provide useful information.

The statistical inference contains all the methods used to evaluate the parameters of a probability model or distribution on observational data (sample) obtained from corresponding “population” [1]. A sample represents a subset of the population, and any parameter evaluated from the samples is necessarily an estimate of the true population parameter, which is generally unknown.

For a given sample, the two most commonly used estimation method are the method of maximum likelihood and the method of moments. This discussion focuses on the second, where the basic concept is that all parameters can be estimated using the information from the moment of the distribution (i.e. mean, variance and so on). Given a sample of size  $n$ , with sample values  $(x_1, x_2, \dots, x_n)$ , the moments of a random variable can be estimated from the samples. In this regard the sample mean and sample variance, which are the main descriptors of a variable, may be defined as the respective averages of the sample values. These are showed in the following equations.

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \quad (\text{A-44})$$

$$s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2 \quad (\text{A-45})$$

Estimating population parameters based on samples naturally raises questions about the accuracy of these estimates. The accuracy improves as  $n$  increases, but there is no direct information about the precision of the estimate without additional measures. To provide quantitative measures of accuracy, confidence intervals  $\alpha$  are used. A confidence interval quantifies the probability that the true parameters value lies within a range, defined by lower and upper limits (two-sided confidence intervals) or by simple upper limit or lower limit (one-sided confidence intervals). One-sided confidence intervals are particularly common in civil engineering applications.

In the following sections, the distribution of the sample mean and standard deviation are discussed for cases with known and unknown variance of the population, focusing on one-sided confidence interval.

#### ESTIMATION OF THE SAMPLES MEAN WITH KNOWN AND UNKNOWN POPULATION VARIANCE

Suppose  $X$  is a random variable whose population variance  $\sigma^2$  is known. For a number of samples  $n$ , the set of sample values  $x_1, x_2, \dots, x_n$  can be considered to have come from a set of independent sample random variables  $X_1, X_2, \dots, X_n$ . Moreover, all previous random variables have the same distribution of population  $X$  with a PDF of  $f_X(x)$ . In this context, the sample mean is itself a random variable. It is denoted as  $\bar{X}$  and can be estimated as

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i \quad (\text{A-46})$$

The expected value of the sample mean is equal to the population mean, indicating it is an unbiased estimator of the population mean  $\mu$  [1].

The variance of the sample mean can be calculated as

$$\text{Var}(\bar{X}) = \frac{\sigma^2}{n} \quad (\text{A-47})$$

If the distribution of  $X$  is Normal, the sample mean  $\bar{X}$  is Normal too. Moreover, as the sample size  $n$  increases, the distribution of the sample mean  $\bar{X}$  becomes narrower (Figure A-3) and better estimation of population mean is obtained.

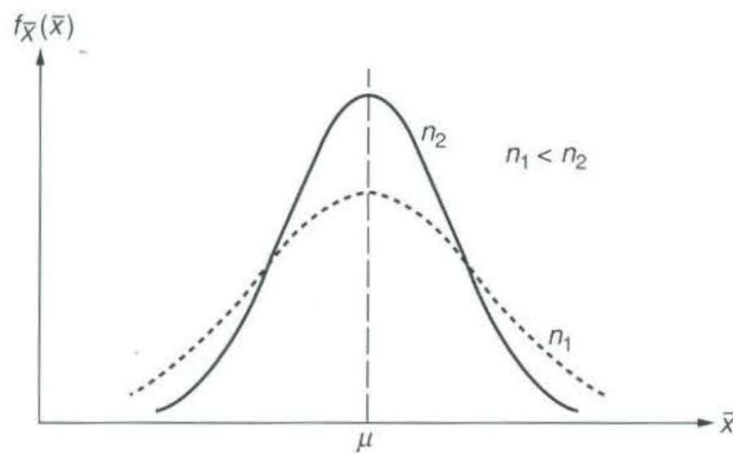


Figure A-3 PDF of sample mean  $\bar{X}$  by varying  $n$  [1]

The probability that the mean fall between a specified lower and upper limit is given by the following expression:

$$P\left(k_{\alpha/2} \leq \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \leq k_{1-\alpha/2}\right) = 1 - \alpha \quad (\text{A-48})$$

where  $1 - \alpha$  is the confidence level, and  $k$  values are the upper and lower values of the standard normal variable evaluated at the probability  $\alpha/2$  and  $1 - \alpha/2$  respectively. From Eq. (A-48), the population mean  $\mu$  having a confidence level of  $1 - \alpha$  can be derived as

$$\langle \mu \rangle_{1-\alpha} = \left( \bar{x} + k_{\alpha/2} \frac{\sigma}{\sqrt{n}}; \bar{x} + k_{1-\alpha/2} \frac{\sigma}{\sqrt{n}} \right) \quad (\text{A-49})$$

In the case of one-sided confidence intervals, i.e. lower limit of the population mean  $\mu$  is obtained as follows

$$\langle \mu \rangle_{1-\alpha} = \bar{x} + k_{1-\alpha} \frac{\sigma}{\sqrt{n}} \quad (\text{A-50})$$

The population variance  $\sigma^2$  is generally unknown and must be substituted by the sample variance  $s^2$ . In this case the distribution of the mean of samples, identified by the random variable  $\frac{\bar{X} - \mu}{s/\sqrt{n}}$ , follows a Student's t distribution, whose PDF is given by

$$f_T(t) = \frac{\Gamma[(f-1)/2]}{\sqrt{\pi f} \Gamma(f/2)} \left(1 + \frac{t^2}{f}\right)^{-(f+1)/2} \quad (\text{A-51})$$

where  $f = n - 1$  is the degrees of freedom. For small value of  $f$ , the PDS of t-distribution has a wider spread than the standard normal one. As  $f$  increases, it tends toward the standard normal distribution as can be seen in Figure A-4.

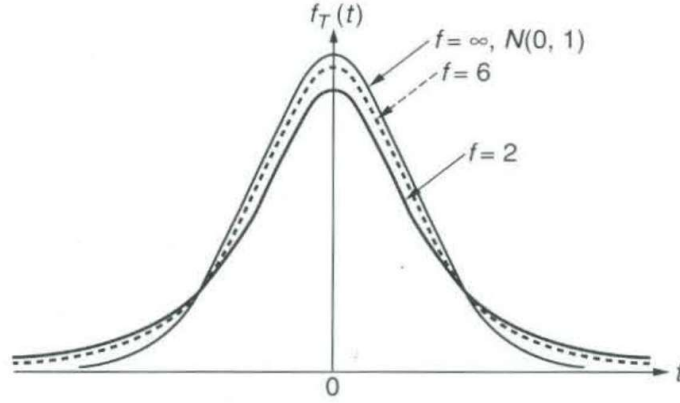


Figure A-4 PDF of  $t$  distribution for varying  $f$  [1]

The probability that the mean fall in a confidence interval is

$$P\left(t_{\alpha/2, n-1} \leq \frac{\bar{X} - \mu}{s/\sqrt{n}} \leq t_{1-\alpha/2, n-1}\right) = 1 - \alpha \quad (\text{A-52})$$

where  $t_{\alpha/2, n-1}$  and  $t_{1-\alpha/2, n-1}$  are, respectively, the lower and the upper of the  $t$  distribution at probability  $\alpha/2$  and  $(1 - \alpha/2)$  with  $f$  d.o.f. The population mean is

$$\langle \mu \rangle_{1-\alpha} = \left( \bar{x} + t_{\alpha/2, n-1} \frac{s}{\sqrt{n}}; \bar{x} + t_{1-\alpha/2, n-1} \frac{s}{\sqrt{n}} \right) \quad (\text{A-53})$$

In the case of one-sided confidence intervals, i.e. lower limit of the population mean  $\mu$  is obtained as follows

$$\langle \mu \rangle_{1-\alpha} = \bar{x} + t_{1-\alpha, n-1} \frac{s}{\sqrt{n}} \quad (\text{A-54})$$

#### ESTIMATION OF THE SAMPLE VARIANCE

Suppose  $X$  is a normal random variable and a number of samples  $n$ , with values  $x_1, x_2, \dots, x_n$ , is collected to estimate the unknown but constant population variance  $\sigma^2$ . As in previous section, these samples may come from independent normal random variables  $X_1, X_2, \dots, X_n$ . The sample variance, that is also a random variable, can be calculated as:

$$S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2 \quad (\text{A-55})$$

The sample variance is an unbiased estimator of the population variance  $\sigma^2$  [1], [2]. The variance of  $S^2$  is given by

$$\text{Var}(S^2) = \frac{\sigma^2}{n} \left( \frac{\mu_4}{\sigma^4} - \frac{n-3}{n-1} \right) \quad (\text{A-56})$$

where  $\mu_4$  is the fourth central moment of the population. It can be observed that as  $n$  increases the sample variance decreases.

By rewriting the Eq. (A-45), the random variable of the sample variance can be identified as  $\frac{(n-1)S^2}{\sigma^2}$ , which is described by  $\chi^2$  distribution with  $f = n - 1$  d.o.f.. If a random variable  $C = \frac{(n-1)S^2}{\sigma^2}$  has a  $\chi^2$  distribution, its PDF is given by

$$f_C(c) = \frac{1}{c^{f/2} \Gamma(f/2)} c^{(f/2-1)} e^{-\frac{c}{2}}, c \geq 0 \quad (\text{A-57})$$

As it is showed in Figure A-5, by increasing  $f$  the  $\chi^2$  distribution tends to approach the normal distribution.

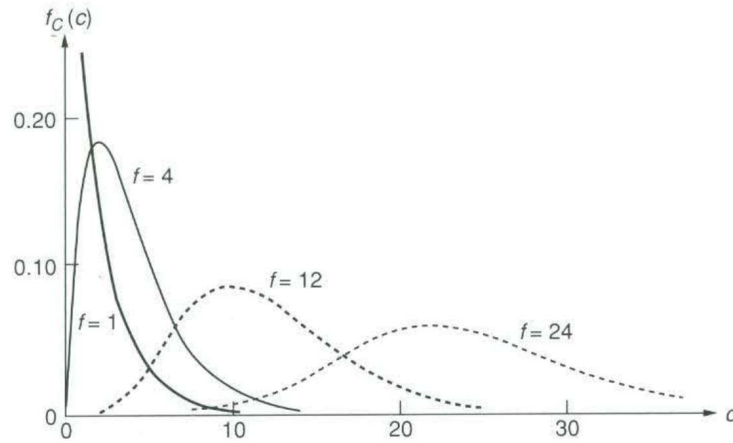


Figure A-5 PDF of  $\chi^2$  distribution by varying  $f$  [1]

The two-sided confidence interval for the population variance  $\sigma^2$  can be expressed as

$$P \left( c_{\alpha/2, n-1} \leq \frac{(n-1)S^2}{\sigma^2} \leq c_{1-\alpha/2, n-1} \right) = 1 - \alpha \quad (\text{A-58})$$

where  $c_{\alpha/2, n-1}$  and  $c_{1-\alpha/2, n-1}$  are, respectively, the lower and the upper of the  $\chi^2$  distribution at probability  $\alpha/2$  and  $(1 - \alpha/2)$  with  $f$  d.o.f. The population variance at two-sided is

$$\langle \sigma^2 \rangle_{1-\alpha} = \left[ \frac{(n-1)s^2}{c_{\alpha/2, n-1}}; \frac{(n-1)s^2}{c_{1-\alpha/2, n-1}} \right] \quad (\text{A-59})$$

In the case of one-sided confidence intervals, i.e. lower limit of  $\sigma^2$  is obtained as follows

$$\langle \sigma^2 \rangle_{1-\alpha} = \frac{(n-1)s^2}{c_{1-\alpha/2, n-1}} \quad (\text{A-60})$$

## POISSON AND RENEWALL PROCESS

The Poisson process is briefly discussed here, while more information can be found in [5]. If the considered reference period is  $[0, T]$ , it can be divided into  $n$  identical sections, where  $n$  is a sufficiently large integer. Correspondingly, the probability of load occurrence within each interval is  $\lambda \frac{T}{n}$ , where  $\lambda$  is the occurrence rate. The probability of occurrence of the  $k$ -th event at time  $t$  follows a Poisson distribution and is given in Eq. (A-26). When a process  $N(t)$  has a Poisson distribution, the interarrival times  $X_k$  ( $k = 1, 2, \dots$ ) are independent and identically distributed according to exponential distribution (Appendix A). It is proven [5] that each interarrival time  $X_k$  is independent and identically distributed exponentially. Thus, a Poisson process has memoryless property.

The Renewal process is a generalization of a stationary Poisson process. In a Poisson process, the time between two successive events follows an exponential distribution independently. If the requirement of exponential distribution is relaxed and counting process where the intervals between two successive events are independently and identically distributed (following any arbitrary continuous distribution) is considered, this generalized process is referred to as a Renewal process. In a Renewal process, the occurrence of an event leads to a "renewal" or reset to a reference (initial) state. Although the state of the process can evolve over time, it always returns to the same reference state when an event occurs. More information about Renewal process can be found in [5].

## REFERENCES

- [1]Ang, A.H. -S, Tang, W.-H., 2007. Probability Concepts in Engineering: Emphasis on Applications in Civil & Environmental Engineering, 2nd ed, Wiley, New York
- [2]Haldar, A., Mahadevan, S., 2000. Probability, reliability, and statistical methods in engineering design. John Wiley, New York; Chichester.
- [3]Evans, M.J., Rosenthal, J.S., n.d. Probability and Statistics: The Science of Uncertainty.
- [4]Billingsley, P., 1995. Probability and measure, 3rd ed. ed, Wiley series in probability and mathematical statistics. Wiley, New York.
- [5]Nakagawa, T., 2011. Stochastic Processes: with Applications to Reliability Theory, Springer Series in Reliability Engineering. Springer London, London.  
<https://doi.org/10.1007/978-0-85729-274-2>