Geometric corrections to cosmological entanglement

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We investigate entanglement production by inhomogeneous perturbations over a homogeneous and isotropic cosmic background, demonstrating that the interplay between quantum and geometric effects can have relevant consequences on entanglement entropy, with respect to homogeneous scenarios. To do so, we focus on a conformally coupled scalar field and discuss how geometric production of scalar particles leads to entanglement. Perturbatively, at first order, we find oscillations in entropy correction, whereas at second order, the underlying geometry induces mode mixing on entanglement production. We thus quantify entanglement solely due to geometrical contribution and compare our outcomes with previous findings. We characterize the geometric contribution through geometric (quasi)particles, interpreted as dark matter candidates.

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I. OVERVIEW

Ascertaining entanglement in curved spacetime remains an outstanding issue of theoretical physics [1–7]. In this respect, the Universe expansion, predicted by Einstein's field equations, can plausibly generate particles [8–10], giving rise to entanglement as due to quantum field evolution in curved spacetime [11–17]. These models mostly assume a homogeneous Friedmann-Robertson-Walker (FRW) background with stationary asymptotic regions in the past and in the future, where the notion of particle, and therefore vacuum, is properly defined. Spacetime expansion is responsible for the fact that asymptotic *in* vacuum is no longer seen as a vacuum state in the *out* region. Instead, it evolves into an *"excited" configuration*, where excitations are interpreted in terms of particle production.[1] Such production from vacuum is quantified through the use of the Bogoliubov coefficients [13].

Quantum correlations arising from particle creation were shown to contain information about the Universe expansion, with strong qualitative differences between bosonic [11] and fermionic fields [12,14,15,17].

The presence of anisotropies also affects entanglement generation [16]. In particular, bosonic and fermionic massless fields only get entangled through anisotropy. This can be of particular relevance for weakly interacting particles like neutrinos, which may have not completely washed out correlations in their evolution to present time. The anisotropic contribution to entanglement is expected to depend upon the direction of particle momentum.

In homogeneous scenarios, the entanglement entropy singles out only particle pairs with opposite momenta. Thus, a natural step to generalize this framework is to include inhomogeneities, which played a fundamental role in the early Universe. Inhomogeneous perturbations departing from a genuine FRW lead to pair-creation probability depending on local geometric quantities [18,19], related to gravitational overdensities, that also modify entanglement production, as gravity becomes locally lumpy. The usual strategy consists in assuming the external field approximation to hold; i.e., the classical perturbed metric is given, and the production of matter fields is studied in this fixed background.

The presence of inhomogeneities implies mode mixing in particle creation, providing then relevant consequences on mode dependence over entanglement measures. We demonstrate that by introducing a position-dependent perturbation, which alters the geometry of the FRW spacetime, scalar particle pairs with different momenta can get entangled. In other words, inhomogeneities and spacetime expansion are both responsible for entanglement

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[1]It is remarkable to stress that cosmic expansion does not modify the spacetime geometry, since it is a consequence of Einstein's equations.

generation, but only inhomogeneities, which we here study for the first time in the framework of cosmological entanglement, allow mode mixing. In this sense, the notion (and the amount) of cosmological entanglement is theoretically enriched with respect to previous efforts in the literature. Moreover, in the limit of negligible expansion, it can be shown that a purely geometric contribution to particle production (and thus to entanglement) is found from local inhomogeneities of a given metric. Accordingly, we propose a novel definition of *geometric cosmological entanglement*,[2] fueled by "geometric" particles, i.e., particles produced by a changing geometry. Geometric particles derive then from the coupling between spacetime perturbations and the scalar field considered. Since we work in the external field approximation, this coupling depends of course on the amount of inhomogeneities prompted by the involved metric.

The limiting case of negligible expansion is discussed in detail both for massive and massless particles. In particular, we show that in a perturbed FRW spacetime massless particles can get entangled only through inhomogeneities, in close analogy with the results of [16] for anisotropies. This suggests that deviations from homogeneous and isotropic models may have relevant consequences on cosmological entanglement and therefore, deserve to be included into the scenario.

In our calculations, we select an asymptotically flat scale factor [9,23] and compute the entanglement entropy up to second order in perturbations. Our goals are (*a*) we investigate perturbatively the quantum and geometric effects on entanglement generation due to inhomogeneities, (*b*) we underline oscillations in particle production at first order, due to the concurrent effects of expansion and geometry, (*c*) we argue how a purely geometric contribution arises at second order and characterize how entanglement changes under the effects of *geometric particles*, (*d*) we interpret dark matter's nature in terms of geometric quasiparticles, fueling (at least partially) the dark matter energy-momentum budget of the Universe.

The manuscript is outlined as follows. In Sec. II, we present the model and describe our perturbative approach to inhomogeneities. In Sec. III, we discuss particle production up to second order. In Sec. IV, we study entanglement generation both for massive and massless particles. In Sec. V, we draw our conclusions. Natural units, $G = \hbar = c = 1$, are here used.

II. SCALAR PARTICLES IN PERTURBED SPACETIMES

We require scalar particles with given mass m nonminimally coupled to spacetime scalar curvature, namely R, perturbing with small inhomogeneities the spatially flat FRW metric, i.e., $g_{\mu\nu} \simeq a^2(\tau)(\eta_{\mu\nu} + h_{\mu\nu})$. Here, $a(\tau)$ is the scale factor, τ the conformal time, $\eta_{\mu\nu}$ the Minkowski metric, and $h_{\mu\nu}$ the small inhomogeneities, $|h_{\mu\nu}| \ll 1$. The scalar field Lagrangian,

$$\mathcal{L} = \frac{1}{2}\sqrt{-g}[g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - (m^2 + \xi R)\phi^2], \tag{1}$$

can be then perturbed up to the first order [18,19] as $\mathcal{L} \simeq \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{O}(h^2)$. Assuming conformal coupling, namely $\xi = 1/6$, the free modes are analytical, enabling the expansion field in terms of *in*, and/or *out*, positive frequency modes by

$$\hat{\phi}(\mathbf{x},\tau) = \frac{1}{a}\int \frac{d^3k}{(2\pi)^{3/2}} \frac{1}{\sqrt{2E_{\mathrm{in}}(k)}} \times [f_k^{\mathrm{in}}(\mathbf{x},\tau)\hat{a}_{\mathrm{in}}(\mathbf{k}) + f_k^{\mathrm{in}*}(\mathbf{x},\tau)\hat{a}_{\mathrm{in}}^{\dagger}(\mathbf{k})], \tag{2}$$

where $E_{\mathrm{in}}(k) = \sqrt{|\mathbf{k}|^2 + m^2a^2(-\infty)}$. The modes f_k can be expressed as $f_k(\mathbf{x},\tau) = f_k(\tau)e^{i\mathbf{k}\cdot\mathbf{x}}$, with $f_k(\tau)$ satisfying

$$\ddot{f}_k(\tau) + [|\mathbf{k}|^2 + m^2a^2(\tau)]f_k(\tau) = 0. \tag{3}$$

Since we now perturb the system, we can work in the interaction picture and express the system final state as

$$\lim_{\tau\to+\infty}|\Psi\rangle = \mathcal{N}\left[|0,\mathrm{in}\rangle + \sum_n \frac{1}{n!}|n,\mathrm{in}\rangle\langle n,\mathrm{in}|S^{(1)}|0,\mathrm{in}\rangle\right], \tag{4}$$

where $S^{(1)} = i\hat{T}\int \mathcal{L}^{(1)}d^4x$ is the first order S matrix. Here, the vector $|n\rangle$ symbolizes any state containing n particles, while $\mathcal{N}$ is a normalizing factor. The interaction Lagrangian is quadratic in the field and its derivatives; thus, particles are produced in pairs up to first order in $h_{\mu\nu}$. The probability amplitude for pair creation is given by the S-matrix element $S_{kp}^{(1)} = \langle\mathbf{kp}|S^{(1)}|0\rangle = i\int\langle\mathbf{kp}|\hat{T}\mathcal{L}^{(1)}|0\rangle d^4x$, where all the states are intended as *in* states. This element gives the transition from the vacuum state $|0\rangle$ to a two-particle state $|\mathbf{kp}\rangle$, with momenta $\mathbf{k}$ and $\mathbf{p}$. An asymptotically flat spacetime is needful to guarantee vacuum uniqueness, so we single out the widely adopted scale factor [18] $a^2(\tau) = A + B\tanh\rho\tau$, with A and B parameters controlling the Universe's volume, while ρ is related to the Universe's expansion rapidity. We suppose the perturbation is not negligible on a finite time interval, namely $\tau \in [\tau_i, \tau_f]$, with τ_i and τ_f negative and $|\tau_i|, |\tau_f| \gg 1$. Accordingly,

$$h_{\mu\nu} = \begin{cases} h_{\mu\nu}, & \text{if } \tau_i < \tau < \tau_f \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

Within this interval, we can safely assume $f_k^{\mathrm{in}}(\tau) \simeq e^{-iE_{\mathrm{in}}\tau}$, provided the expansion is sufficiently fast. Accordingly, in synchronous gauge, $h_{0\nu} = 0$, we can derive the pair creation amplitude [19],

[2] This has not to be confused with geometric measures of entanglement [20–22].

$$S_{kp}^{(1)} = \frac{i\tilde{h}_{\mu\nu}(k+p)}{2(2\pi)^3\sqrt{E_{\rm in}(k)E_{\rm in}(p)}}\left[k^\mu p^\nu - \frac{1}{6}(k+p)^\mu(k+p)^\nu - \frac{1}{12}\eta^{\mu\nu}(k+p)^\sigma(k+p)_\sigma - \frac{1}{2}\eta^{\mu\nu}m^2a^2(\tau\to-\infty)\right]. \quad (6)$$

From the amplitude (6), we can numerically get the amount of created particles. So, it behooves us to thoroughly specify $h_{\mu\nu}$ components. In the synchronous gauge, the metric components g_{00} and g_{0i} are unperturbed, and so

$$ds^2 = a^2(\tau)[d\tau^2 - (\delta_{ij} + h_{ij})dx^i dx^j], \quad (7)$$

where $i,\ j = 1,\ 2,\ 3$. We focus here on scalar modes, portrayed by h and by the traceless part $h_{ij}^{\|}$. Scalar perturbations are easier to compute in *conformal Newtonian gauge*, as the metric is diagonal and one can recognize the gravitational potential, ψ, in Newtonian limit [24]. So, we have

$$ds^2 = a^2(\tau)[(1+2\psi)d\tau^2 - (1-2\phi)dx^i dx_i], \quad (8)$$

where the second scalar, ϕ, is required only if the energy-momentum tensor contains a nonvanishing traceless and longitudinal component. At a first glance, we may set $\psi = \phi$ and consider nearly Newtonian perturbation source, i.e., $\psi = -M/r$, where M is the mass which generates the perturbation and r the radial coordinate. The corresponding scalar perturbation in synchronous gauge, $h_{ij} = h/3 + h_{ij}^{\|}$, can be derived easily following Ref. [24], with the prescription $\dot{a}/a \simeq 0$ in $[\tau_i, \tau_f]$. The details are reported in Appendix A, giving

$$h_{\mu\nu}(x) = -M\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \left[\frac{2}{r} + \left(\frac{3x^2-r^2}{r^5} - \frac{4\pi}{3}\delta(\mathbf{r})\right)\tau^2\right] & 3\tau^2\left(\frac{xy}{r^5}\right) & 3\tau^2\left(\frac{xz}{r^5}\right) \\ 0 & 3\tau^2\left(\frac{xy}{r^5}\right) & \left[\frac{2}{r} + \left(\frac{3y^2-r^2}{r^5} - \frac{4\pi}{3}\delta(\mathbf{r})\right)\tau^2\right] & 3\tau^2\left(\frac{yz}{r^5}\right) \\ 0 & 3\tau^2\left(\frac{xz}{r^5}\right) & 3\tau^2\left(\frac{yz}{r^5}\right) & \left[\frac{2}{r} + \left(\frac{3z^2-r^2}{r^5} - \frac{4\pi}{3}\delta(\mathbf{r})\right)\tau^2\right] \end{pmatrix}. \quad (9)$$

We remark that our assumption of vanishing perturbation outside $[\tau_i, \tau_f]$ can be interpreted in terms of particle backreaction on the spacetime structure [25–30]. In fact, it has been pointed out that the reaction of particle creation back on the gravitational field is able to reduce the creation rate, damping out initial perturbations on timescales of the order of Planck's time. Even if our work is based on the usual external field approximation, a backreaction may in principle justify the transient nature of the perturbation. Bearing $h_{\mu\nu}$ in mind, we can now derive particle production, as we report below.

III. PARTICLE PRODUCTION

At first perturbation order, pair production is due to the combined effect of the expansion and inhomogeneities, whereas at second order the two contributions give instead distinct effects. Hence, *the production rate is nonzero even if the homogeneous background does not produce particles*; i.e., as the *in* and out vacua are identical. At first order, the asymptotic *out* state (4) takes the form,

$$|\Psi\rangle_{\rm out} \equiv \lim_{\tau\to+\infty}|\Psi\rangle = \mathcal{N}\left(|0,{\rm in}\rangle + \frac{1}{2}S_{kp}^{(1)}|\mathbf{kp},{\rm in}\rangle\right), \quad (10)$$

with $\mathcal{N} = 1 + \mathcal{O}(h^2)$ a normalization factor. Introducing Bogoliubov transformations, that relate *in* and *out* ladder operators between them [13], we get the first order number density [18,19],

$$n^{(1)}(k,p) = (2\pi a_f)^{-3}\delta^3(\mathbf{k}+\mathbf{p}){\rm Re}[S_{kp}^{(1)}(\alpha_k^*\beta_k + \alpha_p^*\beta_p)], \quad (11)$$

where $a_f \equiv a(\tau\to+\infty)$ and $\alpha_k,\ \beta_k$ are the Bogoliubov coefficients. It is clear from Eq. (11) that particles are created in pairs with opposite momenta. We introduce now a generic lower bound $r_{\rm min}$ and a factor $e^{-\epsilon r}$, in order to obtain a convergent probability amplitude $S_{kp}^{(1)}$. The parameter ϵ^{-1} can be then interpreted as a cutoff length. This strategy is commonly adopted in field theory, when one consider *regularization* techniques [31]. In such situations, not the whole configuration phase space is considered in view of the fact that the domain is not *a priori* infinite.[3] In our calculations, we set $\epsilon = 1$ for the sake of simplicity, obtaining

$$\tilde{h}_{ij}(2E_{\rm in},0) = -8\pi M e^{-r_{\rm min}}(1+r_{\rm min})\int_{\tau_i}^{\tau_f} e^{2iE_{\rm in}\tau}d\tau\delta_{ij}, \quad (12)$$

[3]Similar approaches are widely used in several contexts, e.g., when dealing with the Casimir effect [32], in renormalization schemes [33], and so on.

Equation (12) can be used to derive the probability amplitude (6) and then the number density (11). We notice that *massless particles are not produced at first order, since* $|\beta_k| = 0$. At second order the number density reads instead

$$n^{(2)}(k,p) = \mathcal{N}^2 (2\pi a_f)^{-3} |S^{(1)}_{kp}|^2 (|\beta_k|^2 + |\beta_p|^2 + 1), \qquad (13)$$

where we exploited the normalization condition $|\alpha_q|^2 - |\beta_q|^2 = 1$ ($q = k,\ p$), and $\mathcal{N}$ is straightforwardly computed from $\langle\Psi|\Psi\rangle = 1$. We notice that *at second order there is a purely geometric contribution*; i.e., particles are produced even if $\beta_q = 0$. As perturbations live where the Universe's expansion is almost negligible, the probability $|S^{(1)}_{kp}|^2$ can be computed explicitly. We separately discuss the massive and massless cases.

A. Massive particles

For slow expansion rate, or Minkowskian background, pair production probability for massive conformally coupled particles is given by

$$\begin{aligned} W^{(1)} &= \int d^3k\, d^3p |S^{(1)}_{kp}|^2 \\ &= \int \frac{d^4q}{(2\pi)^4} \frac{\theta(q^0)\theta(q^2 - 4m^2)}{960\pi} \left(1 - \frac{4m^2}{q^2}\right)^{1/2} \\ &\quad \times \left[\tilde{C}_{\mu\nu\rho\sigma}(q)\tilde{C}^{\mu\nu\rho\sigma}(-q)\left(1 - \frac{4m^2}{q^2}\right)^2 \right. \\ &\quad \left. + \frac{20}{3}\frac{m^4}{q^4}\tilde{R}(q)\tilde{R}(-q)\right], \end{aligned} \qquad (14)$$

where in the last equality, we introduced the four momentum q, i.e., $q = (q^0, \mathbf{q}) = (k^0 + p^0, \mathbf{k} + \mathbf{p})$. In Eq. (14), $\tilde{C}_{\mu\nu\rho\sigma}(q)$ and $\tilde{R}(q)$ are the Fourier transforms of the Weyl tensor and the scalar curvature, respectively. These quantities are derived in Appendix B, as functions of the perturbation tensor $h_{\mu\nu}$. Accordingly, pair production probability can be written in terms of local geometric quantities. In order to compute the $h_{\mu\nu}(x)$ Fourier transform, we assume that the particle momenta are along the z direction, without losing generality, having

$$\tilde{h}_{\mu\nu}(q) = \int d\tau\, e^{iq^0\tau} \int d^3r\, e^{iq_z r\cos\theta} e^{-r} h_{\mu\nu}(x), \qquad (15)$$

where q_z is the total momentum.

Within the framework of massive particles, we can speculate on the fact that the interaction between curvature and scalar field implies a promising scenario toward the existence of geometric quasiparticles of dark matter. Gravitational dark matter production has been recently explored, focusing both on scalar and vector candidates, (see, e.g., Ref. [10] for a recent review). We here conjecture the geometric production enables quasiparticle candidates for dark matter. The scalar field ϕ can be treated as a suitable dark matter contributor [34,35] and, as no interactions are provided with external fields, the field can be constrained according to recent experiments.[4]

Here, we focus on indicative values for m, just to reach our primary goal, i.e., to compare the amount of geometric entanglement with previous findings in homogeneous scenarios. A more realistic interpretation of dark matter in terms of quasiparticles would also require a more meaningful ansatz for the scale factor. For this reason, it will be subject of future investigations that go beyond the purpose of this work.

B. Massless particles

Massless particles can be produced at second order in perturbation, since inhomogeneities break the conformal symmetry of the theory. If $m = 0$, the probability (14) only depends on deviations from conformal flatness, quantified by the Weyl tensor. This result is valid in any FRW spacetime; i.e., it is not strictly request a slowly expanding background.[5]

IV. GEOMETRIC COSMOLOGICAL ENTANGLEMENT

Up to first order, particles are produced in pairs with opposite momenta; i.e., there is no mode mixing. We write the *in* vacuum in a Schmidt decomposition of *out* states [11],

$$|0_k; 0_{-k}\rangle_{\text{in}} = \sum_{n=0}^{\infty} c_n |n_k; n_{-k}\rangle_{\text{out}}, \qquad (16)$$

where c_n is a normalization constant and n_k labels the number of excitations in the field mode k as seen by an inertial observer in the *out* region. The *in* state containing one pair,

$$|1_k; 1_{-k}\rangle_{\text{in}} = \hat{a}^\dagger_{\text{in}}(\mathbf{k})\hat{a}^\dagger_{\text{in}}(-\mathbf{k})|0_k; 0_{-k}\rangle_{\text{in}}, \qquad (17)$$

can be written in the *out* region exploiting the inverse Bogoliubov transformations, as described in Appendix C. To evaluate the entanglement amount, we consider the first order density operator $\rho^{\text{out}}_{k,-k} = |\Psi\rangle_{\text{out}}\langle\Psi|$ and Eq. (10) with $\mathbf{p} = -\mathbf{k}$. Performing a partial trace over one subsystem, we obtain the reduced density operator,

[4]For instance, in previous efforts, a benchmark mass around $m = 1$ MeV can be assumed for geometric dark matter contribution [34,36,37].

[5]This is due to the fact that the Weyl tensor $C^\mu{}_{\nu\rho\sigma}(x)$ is invariant under conformal transformations.

$$\rho_k^{\rm out} = {\rm Tr}_{-k}(\rho_{k,-k}^{\rm out}). \tag{18}$$

The coefficients c_n are derived from the normalization $\langle\Psi|\Psi\rangle_{\rm out} = 1$. Introducing the quantity $\gamma = |\beta_k^*/\alpha_k|^2$, after some algebra, we arrive to the final expression for the density operator,

$$\rho_k^{\rm out} = \frac{(1-\gamma)^2}{1-\gamma+(1+\gamma){\rm Re}(S_{k,-k}^{(1)}\alpha_k^*\beta_k)} \times \sum_{n=0}^{\infty}\gamma^n(1+{\rm Re}(S_{k,-k}^{(1)}\alpha_k^*\beta_k)(2n+1))|n\rangle_k\langle n|. \tag{19}$$

The corresponding von Neumann entropy is then

$$\mathcal{S}(\rho_k^{\rm out}) = -{\rm Tr}(\rho_k^{\rm out}\log_2\rho_k^{\rm out}) = -\sum_{n=0}^{\infty}\lambda_n\log_2\lambda_n, \tag{20}$$

where λ_n are the $\rho_k^{\rm out}$ eigenvalues, say

$$\lambda_n = \frac{(1-\gamma)^2}{1-\gamma+(1+\gamma){\rm Re}(S_{k,-k}^{(1)}\alpha_k^*\beta_k)} \times \gamma^n(1+{\rm Re}(S_{k,-k}^{(1)}\alpha_k^*\beta_k)(2n+1)). \tag{21}$$

We notice that in the limit of vanishing perturbation ($S_{k,-k}^{(1)} = 0$) the well-known zero order eigenvalues are recovered,

$$\lambda_n^{(0)} = (1-\gamma)\gamma^n, \tag{22}$$

leading to the usual expression for the entropy [11],

$$\mathcal{S}^{(0)} = \log_2\left(\frac{\gamma^{\gamma/(\gamma-1)}}{1-\gamma}\right). \tag{23}$$

In Fig. 1, we show the *entropy shift*, $(\mathcal{S}-\mathcal{S}^{(0)})/\mathcal{S}^{(0)}$, for given values of A. The perturbation parameters are chosen so that $|h_{\mu\nu}(x)| \ll 1$, with the time parameters sufficiently large to fulfill the requirements of Sec. II. Remarkably, the first order contribution turns out to be quite small, whereas entanglement entropy oscillates. This means that *kinds of expansion and spacetime geometry can, in principle, decrease the amount of entanglement*, turning to be more relevant for small Universe volumes, i.e., small A. On the other hand, the correction is larger at small momenta, due to the fact that at first order particles are mainly produced as k is small. We expect first order corrections to be more relevant if the hypothesis of spherically symmetric perturbation is released.

We now turn to second order, where particles are produced in pairs with generic momenta k and p, with a robust purely geometric contribution. The corresponding entanglement generation is summarized below.

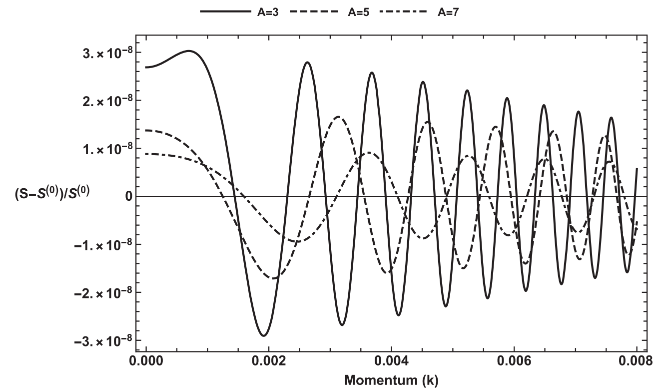

FIG. 1. Entropy shift as function of the momentum k, for different values of the parameter A. The other parameters are $B=2, \rho=1, m=0.01, n=40, M=10^{-5}, r_{\rm min}=5, \tau_i=-10^4$, and $\Delta\tau \equiv \tau_f - \tau_i = 100$.

A. Massive particles

Here, $n^{(2)}(k,p)$ is mainly settled by $\propto |C_{\mu\nu\rho\sigma}(q)|^2$ and $\propto |R(q)|^2$, as evident from Eq. (14). It is then possible to set $\beta_k = \beta_p = 0$, neglecting expansion effects, getting the state (10) in terms of the *out* basis as

$$|\Psi\rangle = \mathcal{N}\left(|0_k;0_p\rangle + \frac{1}{2}S_{kp}^{(1)}|1_k;1_p\rangle\right), \tag{24}$$

where we remove the *out* subscript since the *in* and *out* vacua coincide if the β coefficients vanish. We remark that second order S-matrix elements can be neglected in Eq. (24). This happens because the interacting Lagrangian is still quadratic in the field at second order in perturbations, and no $|00\rangle$ components for $S^{(2)}$ have been found in our computation. Hence, this implies that particles are again produced at least as pairs. Any further order would therefore contribute to the density operator from third or higher perturbative order and may be easily neglected. Equation (24) shows up a bipartite pure state. In order to quantify the corresponding entanglement entropy, we trace out the "p" or "k" contribution. Accordingly, we are left with the following reduced density operator:

$$\rho_k = \mathcal{N}^2\left(|0\rangle_k\langle 0| + \frac{1}{4}|S_{kp}^{(1)}|^2|1\rangle_k\langle 1|\right), \tag{25}$$

where the probability of pair production $|S_{kp}^{(1)}|^2$ is given by Eq. (14). The subsystem entropy, following from Eq. (25), is plotted in Fig. 2 as function of k. Inhomogeneous perturbations break space translation symmetry; thus, linear momentum is no longer conserved in particle creation processes. Accordingly, *second order entropy is characterized by notable mode mixing*. This is an impressive property of geometric cosmological entanglement, never put forward. We also notice that second order corrections to entanglement

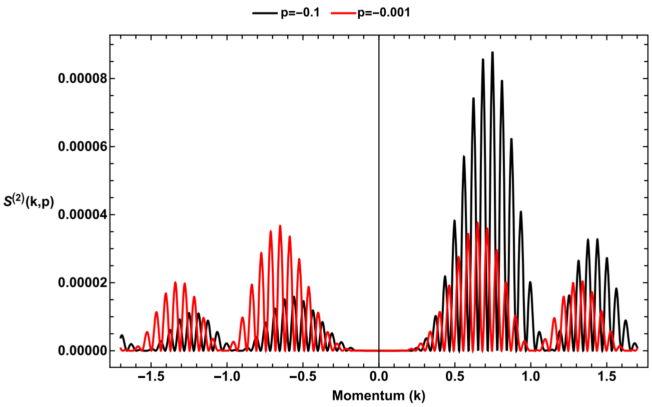

FIG. 2. Entanglement entropy $\mathcal{S}^{(2)}(k,p)$ as function of the particle momentum k, with $p = -0.1$ and $p = -0.001$. The other parameters are the same of Fig. 1, with $A = 3$.

are typically larger than first order ones. As anticipated, this effect is related to our choice of a spherically symmetric perturbation, which makes the first order contribution negligible.

B. Massless particles

Here, $\beta_k = \beta_p = 0$ is naturally fulfilled, due to the conformal symmetry. Starting from the state in Eq. (24), the reduced density operator takes again the form (25), and pair creation probability is determined by the Weyl tensor only. In Fig. 3, we display the entanglement entropy as function of the momentum k, assuming the same parameters as in the massive case. The amount and mode dependence of entanglement closely resemble the massive case, thus showing that geometric effects on entanglement production turn out to be similar in both the aforementioned cases.

V. OUTLOOKS

We computed the entanglement amount in curved spacetime as due to inhomogeneous perturbations over a homogeneous FRW and confronted our findings with previous results developed in the literature. In particular, we demonstrated how geometric cosmological entanglement may arise in the framework of negligible spacetime expansion, i.e., when particles are produced only due to perturbations of the underlying spacetime geometry. Geometric entanglement is thus interpreted as due to geometric quasiparticles that arise from the interacting contribution between geometry and fields. We also discussed the main differences in entanglement generation between massive and massless scalar particles. We then expect the matching between quantum and geometric effects drastically alter the entanglement measures. From the one hand, we showed at first perturbative order oscillations in entropy corrections occurred, while at second order a non-negligible entropy, featured by non-vanishing mode mixing, arises in entanglement generation. To interpret this scheme, we noticed geometrical dark matter can be reviewed as a suitable benchmark scenario where *dark matter emerges under the form of quasiparticles*. We proposed ϕ as a suitable dark matter contributor since no interactions are provided with external fields, discussing possible mass values, in agreement with recent observations. This paves the way for further investigations deepening the interconnections between the notions of entanglement and dark matter. According to our findings, we also expect geometry to exhibit relevant consequences on entanglement extraction protocols, e.g., entanglement harvesting. Consequently, this would open novel strategies to deduce cosmological parameters, to interpret dark matter under a new promising way, or to depict primordial quantum gravity stages.

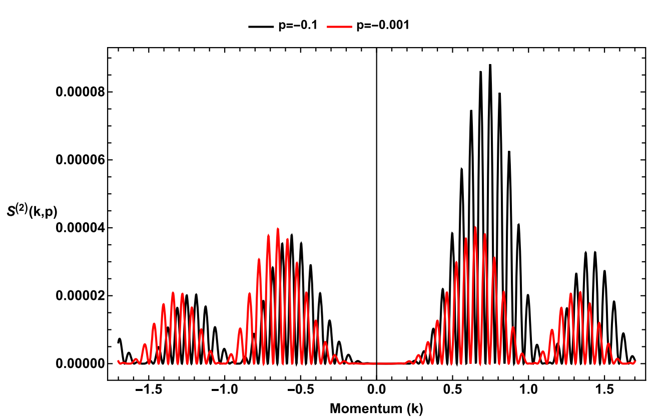

FIG. 3. Entanglement entropy $\mathcal{S}^{(2)}(k,p)$ for massless particles, as function of the momentum k with $p = -0.1$ and $p = -0.001$. The other parameters are the same as in Fig. 1, with $A = 3$.

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APPENDIX A: SCALAR PERTURBATIONS IN THE SYNCHRONOUS AND CONFORMAL NEWTONIAN GAUGES

We here briefly recall the gauge transformation between the synchronous and conformal Newtonian gauges [24]. We focus on scalar perturbations only. Let us consider a general coordinate transformation from a system x^μ to another $\hat{x}^\mu$,

$$x^\mu \to \hat{x}^\mu = x^\mu + d^\mu(x^\nu). \tag{A1}$$

We write the time and the spatial parts separately as

$$\hat{x}^0 = x^0 + \alpha(\mathbf{x}, \tau) \tag{A2}$$

$$\hat{\mathbf{x}} = \mathbf{x} + \vec{\nabla}\beta(\mathbf{x},\tau) + \boldsymbol{\epsilon}(\mathbf{x},\tau), \qquad \vec{\nabla}\cdot\boldsymbol{\epsilon} = 0, \tag{A3}$$

where the vector d has been divided into a longitudinal component $\vec{\nabla}\beta$ and a transverse component $\vec{\epsilon}$. Let $\hat{x}^\mu$ denote the synchronous coordinates and x^μ the conformal Newtonian coordinates, with $\hat{x}^\mu = x^\mu + d^\mu$. It is easy to see that

$$\alpha(\mathbf{x},\tau) = \dot{\beta}(\mathbf{x},\tau), \tag{A4}$$

$$\epsilon_i(\mathbf{x},\tau) = \epsilon_i(\mathbf{x}), \tag{A5}$$

$$h^{\parallel}_{ij}(\mathbf{x},\tau) = -2\left(\partial_i\partial_j - \frac{1}{3}\delta_{ij}\nabla^2\right)\beta(\mathbf{x},\tau), \tag{A6}$$

$$\partial_i\epsilon_j + \partial_j\epsilon_i = 0, \tag{A7}$$

and

$$\psi(\mathbf{x},\tau) = -\ddot{\beta}(\mathbf{x},\tau) - \frac{\dot{a}}{a}\dot{\beta}(\mathbf{x},\tau), \tag{A8}$$

$$\phi(\mathbf{x},\tau) = +\frac{1}{6}h(\mathbf{x},\tau) + \frac{1}{3}\nabla^2\beta(\mathbf{x},\tau) + \frac{\dot{a}}{a}\dot{\beta}(\mathbf{x},\tau). \tag{A9}$$

Assuming an asymptotically flat spacetime as described in Sec. II, we can safely conclude that $\dot{a}/a \simeq 0$ in the region $[t_i, t_f]$. Accordingly, (A8) would give

$$\beta(r,\tau) \simeq +\frac{M}{2r}\tau^2. \tag{A10}$$

Subtracting now (A9) from (A8), with the assumption $\psi = \phi$, we obtain

$$h(r,\tau) = -6\ddot{\beta} - 2\nabla^2\beta = -2M\left[\frac{3}{r} - 2\pi\tau^2\delta(\mathbf{r})\right]. \tag{A11}$$

Finally, from (A6), we get

$$h^{\parallel}_{ij}(r,\tau) = -M\tau^2\left[\partial_i\partial_j\left(\frac{1}{r}\right) - \frac{1}{3}\delta_{ij}(-4\pi\delta(\mathbf{r}))\right]. \tag{A12}$$

Recalling the well-known result,

$$\partial_i\partial_j\left(\frac{1}{r}\right) = \frac{3x_ix_j - r^2\delta_{ij}}{r^5} - \frac{4\pi}{3}\delta(\mathbf{r})\delta_{ij}, \tag{A13}$$

and the expression for scalar perturbations in synchronous gauge, we finally arrive at the tensor $h_{\mu\nu}$, namely Eq. (9).

APPENDIX B: CURVATURES AND WEYL TENSOR IN LINEARIZED GRAVITY

As discussed in Sec. III, pair production probability at second order depends on local geometric quantities. Here, we recall the main results from linearized gravity, which are useful in order to derive the probability of pair creation (14). Starting from the perturbed Minkowski metric,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(x), \tag{B1}$$

the connection coefficients are

$$\Gamma^{\rho}{}_{\rho\mu\nu} = \frac{1}{2}\eta^{\rho\sigma}(\partial_\mu h_{\nu\sigma} + \partial_\nu h_{\mu\sigma} - \partial_\sigma h_{\mu\nu}). \tag{B2}$$

Accordingly, the first order Riemann curvature is

$$\begin{aligned} R_{\mu\nu\rho\sigma} &= \eta_{\mu\lambda}(\partial_\sigma\Gamma^{\lambda}{}_{\lambda\nu\rho} - \partial_\rho\Gamma^{\lambda}{}_{\lambda\nu\sigma}) \\ &= \partial_\rho\partial_{[\nu}h_{\mu]\sigma} + \partial_\sigma\partial_{[\mu}h_{\nu]\rho}, \end{aligned} \tag{B3}$$

where square brackets denote antisymmetrization, as usual. The Ricci curvature follows as

$$R_{\mu\nu} = R^{\rho}_{\mu\rho\nu} = \frac{1}{2}\partial^\rho\partial_\rho h_{\mu\nu} - \partial^\rho\partial_{(\mu}h_{\nu)\rho} + \frac{1}{2}\partial_\mu\partial_\nu h, \tag{B4}$$

and the Ricci scalar as

$$R = \eta^{\mu\nu}R_{\mu\nu} = \partial^\mu\partial_\mu h - \partial^\mu\partial^\nu h_{\mu\nu}. \tag{B5}$$

Introducing now the Fourier transform of the perturbation,

$$\tilde{h}_{\mu\nu} = \int d^4q\, e^{iqx}h_{\mu\nu}(x), \tag{B6}$$

it is straightforward to obtain

$$\tilde{R}(q) = q^\mu q^\nu \tilde{h}_{\mu\nu}(q) - q^2\tilde{h}(q). \tag{B7}$$

Assuming a real perturbation, one finds

$$\begin{aligned} |R(q)|^2 &= R(q)R(-q) \\ &= q_\mu q_\nu q_\rho q_\sigma h^{\mu\nu}(q)h^{\rho\sigma}(-q) + q^4 h(q)h(-q) \\ &\quad - q^2 q_\mu q_\nu[h^{\mu\nu}(q)h(-q) + h^{\mu\nu}(q)h(-q)], \end{aligned} \tag{B8}$$

$$\begin{aligned} |R_{\mu\nu}(q)|^2 = &\frac{1}{2}q_\mu q_\nu q_\rho q_\sigma h^{\mu\rho}(q)h^{\nu\sigma}(-q) \\ &-\frac{q^2}{2}q_\rho q^\mu h_{\mu\nu}(q)h^{\nu\rho}(-q) + \frac{1}{4}q^4 h^{\mu\nu}(q)h_{\mu\nu}(-q) \\ &-\frac{1}{4}q^2 q_\mu q_\rho[h^{\mu\rho}(q)h(-q) + h^{\mu\rho}(-q)h(q)], \end{aligned} \tag{B9}$$

$$\begin{aligned} |R_{\mu\nu\rho\sigma}|^2 = &\frac{1}{4}q^4 h^{\mu\nu}(q)h_{\mu\nu}(-q) - 2q^2 q_\mu q^\nu h_{\nu\sigma}(q)h^{\mu\sigma}(-q) \\ &+ q_\mu q_\nu q_\rho q_\sigma h^{\mu\rho}(q)h^{\nu\sigma}(-q). \end{aligned} \tag{B10}$$

From Eqs. (B8)–(B10), it can be shown that

$$|R_{\mu\nu\rho\sigma}|^2 - 4|R_{\mu\nu}|^2 + |R|^2 = 0. \tag{B11}$$

In four dimensions, the Weyl conformal tensor takes the form,

$$C_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} + \frac{1}{3} R g_{\mu[\sigma} g_{\rho]\nu} + (g_{\nu[\rho} R_{\sigma]\mu} - g_{\mu[\rho} R_{\sigma]\nu}), \tag{B12}$$

that gives

$$\begin{aligned} |C_{\mu\nu\rho\sigma}(q)|^2 &= C_{\mu\nu\rho\sigma}(q) C^{\mu\nu\rho\sigma}(-q) \\ &= |R_{\mu\nu\rho\sigma}|^2 - 2|R_{\mu\nu}|^2 + \frac{1}{3}|R|^2. \end{aligned} \tag{B13}$$

Now, exploiting Eq. (B11), we can rewrite Eq. (B12) as

$$|C_{\mu\nu\rho\sigma}(q)|^2 = 2|R_{\mu\nu}|^2 - \frac{2}{3}|R|^2. \tag{B14}$$

The probability of pair production at second order in the perturbation, both for massive and massless particles, can be computed starting from the result (B14), as discussed in [18,19].

APPENDIX C: FIRST ORDER DENSITY OPERATOR

Here, we derive the explicit form of the reduced density operator (18), which is required in order to quantify first order corrections to entanglement entropy. In the *out* region, the two-particle state (17) reads

$$\begin{aligned} |1_k; 1_{-k}\rangle_{\text{in}} &= (\alpha_k^* \hat{a}_{\text{out}}^\dagger(\mathbf{k}) + \beta_k \hat{a}_{\text{out}}(-\mathbf{k}))(\alpha_k^* \hat{a}_{\text{out}}^\dagger(-\mathbf{k}) + \beta_k \hat{a}_{\text{out}}(\mathbf{k})) \sum_{n=0}^{\infty} c_n |n_k; n_{-k}\rangle_{\text{out}} \\ &= (\alpha_k^*)^2 \sum_{n=0}^{\infty} (n+1) c_n |n+1; n+1\rangle_{\text{out}} + \alpha_k^* \beta_k \sum_{n=0}^{\infty} n c_n |n_k; n_{-k}\rangle_{\text{out}} \\ &\quad + \alpha_k^* \beta_k \sum_{n=0}^{\infty} (n+1) c_n |n_k; n_{-k}\rangle_{\text{out}} + \beta_k^2 \sum_{n=0}^{\infty} n c_n |n-1; n-1\rangle_{\text{out}}, \end{aligned} \tag{C1}$$

where we have exploited the Bogoliubov transformations relating asymptotic ladder operators, which has the general form [13],

$$\hat{a}_{\text{out}}(\mathbf{k}) = \alpha_k^* \hat{a}_{\text{in}}(\mathbf{k}) - \beta_k^* \hat{a}_{\text{in}}^\dagger(-\mathbf{k}), \tag{C2}$$

$$\hat{a}_{\text{in}}(\mathbf{k}) = \alpha_k \hat{a}_{\text{out}}(\mathbf{k}) + \beta_k^* \hat{a}_{\text{out}}^\dagger(-\mathbf{k}). \tag{C3}$$

From the state (10) and the expansion (16), we can then write the density operator $\rho_{k,-k}^{(\text{out})} = |\Psi\rangle_{\text{out}}\langle\Psi|$ up to first order, and performing a partial trace over antiparticles, we are left with

$$\begin{aligned} \rho_k^{\text{out}} &= \text{Tr}_{-k}(\rho_{k,-k}^{\text{out}}) \\ &= \sum_{n=0}^{\infty} |c_n|^2 |n\rangle_k \langle n| + \frac{1}{2} S_{k,-k}^{(1)} \alpha_k^* \beta_k \left[\sum_{n=0}^{\infty} (n+1)|c_n|^2 |n\rangle_k \langle n| + \sum_{n=0}^{\infty} n |c_n|^2 |n\rangle_k \langle n| \right] \\ &\quad + \frac{1}{2} S_{k,-k}^{(1)*} \alpha_k \beta_k^* \left[\sum_{n=0}^{\infty} (n+1)|c_n|^2 |n\rangle_k \langle n| + \sum_{n=0}^{\infty} n |c_n|^2 |n\rangle_k \langle n| \right]. \end{aligned} \tag{C4}$$

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