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Resilience enhancement to loss of actuator effectiveness in a Model-Free Adaptive framework

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ABSTRACT

Within the theoretical framework supporting the Model Free Adaptive Control approach, this paper investigates on the presence of possible loss of effectiveness affecting the actuator of a Single-Input Single-Output plant. In particular, stemming from a tracking controller proposed in a seminal paper and the related proofs, an identification algorithm is proposed for the estimation of the loss of effectiveness, with proved bounded estimation error, along with a condition for the detection of actuator faults. A fault-resilient tracking controller is finally proposed, based on the estimated loss of effectiveness, providing asymptotically vanishing tracking error. A comparative analysis by simulation on a benchmark system taken from the pertinent literature is also presented to validate the proposed development.

1. Introduction

Resilience with regard to possible efficiency losses or failures of system components is a central requirement for control design regardless of the chosen plant description, the technique used for design and the control problem addressed. In fact, all the established Model-Based (MB) methodologies which are based on a (relatively) accurate modeling of the plant to be controlled share the tasks to be pursued by the control system with alternative techniques, which are rapidly evolving, such as the so called Data-Driven Control (DDC) methods [1]. In the Data-Driven (DD) framework, the controller is designed by directly using online or offline Input/Output (I/O) data coming from the controlled system, or using knowledge from data processing. Not surprisingly, given the increasing complexity of industrial processes and the easy availability of large amounts of measured process data made possible by information technology, this approach is more and more attracting the interest of control researchers.

DDC approaches can be classified according to the type of data used (online/offline) or the controller structure (known/unknown) [2]. Offline methods, such as Iterative Feedback Tuning (IFT) [3], Virtual Reference Feedback Tuning (VRFT) [4], use offline data to search for a consistent dynamic model, which is then used for the controller design. The resulting controller is usually very efficient and precise, but shows poor adaptability. In contrast, online DDC methods, such as Model Free Adaptive Control (MFAC) [5], Unfalsified Control [6], can easily adapt to changes in the process as real-time data is used to continuously update the system and controller parameters, but at the cost of lower computational efficiency and promptness of response. Iterative Learning Control (ILC) is considered a hybrid offline/online DDC and shows high precision in repetitive tasks. Recently, several hybrid approaches have been proposed, e.g. [7], which combines IFT algorithms and fuzzy control, [8], which couples Adaptive Dynamic Programming (ADP), state reconstruction and output feedback for the control of a hydraulic servo actuator, [9], where an adaptive neural finite-time dynamic surface control strategy is proposed for a category of fractional-order nonlinear large-scale systems. In particular, MFAC has been extensively used and combined with many control approaches [2,10,11], due to its appealing design principles associated with the introduction of an equivalent dynamic linearization model based on Pseudo-Partial Derivatives (PPDs), which simplify

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the model structure and the resulting control design [2,5,12]. Significant efforts are currently being made to extend the scope of application of MFAC to cases where state stability is required in addition to I/O stability (see e.g. [13] and references therein).

In the MB framework, research on fault detection, estimation, accommodation is nowadays well established to a great extent (see [14,15] and references therein). In the DD context, on the contrary, the topic is currently under investigation, though DD fault tolerant design methods began to appear in the literature starting from more than a decade ago [16]. It should be noticed that in a number of recent papers, an unknown but linear model structure for the plant is assumed. For instance, a DD method for designing the LQR has been latterly published in [17], while a DD fault-resilient controller for discrete-time plants subject to actuator fault has been proposed in [18], with data-based LMIs providing stability conditions. Less recently, an integrated DD approach to fault-resilient control for MIMO systems has been presented in [19], while a novel DD control structure has been presented [20] in order to find a middle ground between robustness and performance in the traditional feedback framework. An online fault detection and isolation strategy using a symbolic based linear multi-model concept for gas turbine engines is discussed in [21].

Due to the inherently discrete nature of the available process data in the DD framework, the adoption of discrete-time, adaptive, equivalent representations of the unknown plant can be an appropriate option for investigating resilient control methods for a wider class of processes. The MFAC framework has been in fact adopted in [22] considering the presence of possible sensor faults and adopting an Neural Network (NN) to capture fault dynamics and perform control redesign. Analogous approaches have been pursued in [23,24] with reference to a nonlinear SISO Model Free (MF) system in the presence of sensor faults, adopting NN-based fault estimation schemes. Very recently, speed sensor faults affecting subway train speed tracking controllers have been considered in [25], with train dynamics transformed into a dynamic linearization data model and with the fault function approximated by a NN, accommodated by an adaptive ILC scheme that also considers traction/braking force constraints. Turning to the case when actuator faults are considered, in view of their recognized effect as source of performance degradation (even of instability), the number of results available in the DD literature remarkably decreases. In addition to [18] where an LMI-based policy is proposed, ADP-based Fault Tolerant Control (FTC) approaches have been developed in recent years (see f.i. the FTC problem of a hydraulic servo actuator in the presence of actuator faults studied in [26] and references therein).

The present work fits the theoretical framework supporting the MFAC approach, and addresses the output regulation problem for a Single-Input Single-Output (SISO) nonlinear plant possibly affected by Loss of Effectiveness (LoE) of the actuator. The numerous successful applications in various areas testify to the attractiveness of MFAC for practical use due to its recognized advantages, associated to the use of PPDs which are robust to the time variability of system parameters, structure and time delay [13]. With respect to the references previously cited, the computational simplicity of MFAC, is indeed a further significant advantage in practical implementation. In the case of actuator faults, the adaptivity features of MFAC could be expected to inherently compensate, to some extent, for faults affecting the plant, but the is no denying that a deeper investigation could provide a more accurate quantification of the achievable resilience. It should be emphasized that this work extends a previous study [27] that considered Unmanned Ground Vehicles for precision agriculture with the purpose of mitigating the loss of effectiveness of actuators associated with slipping, relaxing a number of assumptions and dealing with a general class of plants. In this paper:

- The MFAC framework and the associated proofs, originally proposed in [5] are investigated in the presence of possible loss of actuator effectiveness;
- an estimation algorithm is presented for the LoE, with proved closed-loop bounded estimation error;
- a fault-resilient tracking controller for MF discrete-time systems is finally proposed, obtained by a rearrangement of the controller exploiting the LoE's estimate, proved able to reinforce the inherent recovery ability of the adaptive scheme.

In the following sections, details about the considered plant and the addressed control problem will be preliminarily given (Sections 2 and 3). Section 4 contains the proposal of a fault estimation algorithm and a tracking control law fault-resilient with respect to possible LoE of the actuator, along with a condition for fault detection. Results coming from a comparative simulation study will be reported in Section 5 with reference to a benchmark system borrowed from the DD literature, considering both offline and online DDS algorithms. The achieved performances, along with pro and cons of the analyzed techniques, will be there discussed, and conclusions will be drawn in Section 6. The following symbols and notation will be used in the paper, and are here listed for convenience. $\mathbb{R}$ denotes as usual real numbers, $\mathbb{R}^n$ is the n-dimensional space, $\|\cdot\|$ the Euclidean norm. Input and output variables at time instants $k \in \mathbb{N}_0$ are denoted as $u(k) \in \mathbb{R}$ and $y(k) \in \mathbb{R}$, and $\delta(k) \in \mathbb{R}$ represents the loss of actuator effectiveness.

2. Preliminaries

Consider the following general description of a SISO discrete-time nonlinear non-affine system:

$$y(k+1) = f(y(k), \dots y(k-n_y), u(k), \dots, u(k-n_u)) \tag{1}$$

where $u(k) \in \mathbb{R}$ and $y(k) \in \mathbb{R}$ are the I/O variables at time instants $k \in \mathbb{N}_0$. The integers $n_y, n_u \in \mathbb{Z}_+$ are unknown orders, and $f(\cdot) : \mathbb{R}^{nu+ny+2} \to \mathbb{R}$ is an unknown nonlinear function. With reference to a discrete-time variable $z(k)$ of order n_z, the difference between two consecutive values will be defined hereafter as $\Delta z(k-i) \triangleq z(k-i) - z(k-i-1)$, while a corresponding vector $\Delta \mathbf{Z}(k-\nu) \in \mathbb{R}^{d_z}$ is defined containing d_z past samples of the same variable, i.e., $\Delta \mathbf{Z}(k-\nu) \triangleq \begin{bmatrix} \Delta z(k-\nu), & \Delta z(k-\nu-1), \dots, \Delta z(k-\nu-d_z+1) \end{bmatrix}^T$, with $\nu \in \mathbb{N}_0$, $z(k) = 0$ for $k \leq 0$.

An I/O data-based model, completely equivalent to the original model (1) in I/O terms, has been shown [2,12] to be derivable adopting the Partial Form Dynamic Linearization (PFDL). No explicit or implicit structure/parameter information is assumed about plant dynamics except for the following two general Assumptions: (i) the partial derivatives of $f(\dots)$ with respect to the control input $u(k)$, $u(k-1)$, … are assumed continuous, and (ii) the original plant is assumed generalized Lipschitz (i.e., $|\Delta y(k+1)| \leq L\|\Delta \mathbf{U}(k)\|$, $\|\Delta \mathbf{U}(k)\| \neq 0||$), meaning that the output change corresponding to a finite input change cannot be infinite.

The following Theorem formally introduces the Pseudo Partial Derivative (PPD) based model of the plant (1).

Theorem 2.1 ([5]). *For the nonlinear system (1) satisfying the previous Assumptions, for any fixed pseudo order $d_u > 0$ there exists a parameter vector $\mathbf{\Phi}(k-\nu)$, called the PPD vector, such that the plant can be transformed into the following equivalent PFDL description*

$$\Delta y(k+1) = \mathbf{\Phi}(k-\nu)^T \Delta \mathbf{U}(k-\nu) \tag{2}$$

where $\mathbf{\Phi}(k-\nu) = \left[\begin{array}{c} \phi_1(k-\nu), \phi_2(k-\nu), \dots, \phi_{du}(k-\nu) \end{array}\right]$, and $\|\mathbf{\Phi}(k-\nu)\| \leq b$ where b is a positive constant.

Without loss of generality, for the sake of simplicity of notation, $\nu = 0$ will be set hereafter.

Remark 2.1. The PFDL description (2), though apparently restrictive, is able to represent a large class of systems. As argued in [5], consider for instance a linear FIR system of the form $y(k) = H(q^{-1})u(k)$, with q^{-1} being the standard backshift operator, possibly coming from an identification procedure. The plant model can be rewritten as $\Delta y(k) = H(q^{-1})\Delta u(k)$, corresponding to (2) with possibly time-varying FIR coefficients.

In the present set-up, the presence of a possible loss of actuator effectiveness is considered, starting from an unknown time instant $\bar{k} > 0$, with respect to the theoretically applied control law:

$$u^F(k) = \begin{cases} u(k) & k < \bar{k} \\ \delta(k)u(k) & k \geq \bar{k} \end{cases} \tag{3}$$

with $0 < \delta_m < \delta(k) \leq 1$, $\forall k$.

Define $\mathbf{\Delta}(k) \triangleq diag\left\{\delta(k), \delta(k-1), \dots, \delta(k-d_u+1)\right\}$. Under the following assumption:

Assumption 2.1. It is assumed that the fault $\delta(k)$ is slowly varying with respect to the input variable, i.e., $\Delta\delta(k)u(k-1) \simeq 0$, $\forall k$, this implying:

$$\delta(k)u(k) - \delta(k-1)u(k-1) \simeq \delta(k)\Delta u(k) \tag{4}$$

the PPD model (2) possibly accounting for a faulty condition can be rewritten as

$$\Delta y(k+1) = \mathbf{\Phi}(k)^T \mathbf{\Delta}(k) \Delta \mathbf{U}(k) \tag{5}$$

where the estimation algorithm of the estimated value $\hat{\mathbf{\Phi}}(k)$ the unknown PPD vector $\mathbf{\Phi}(k)$ proposed by [5] can be still applied:

$$\begin{aligned} &\hat{\mathbf{\Phi}}(k) = \hat{\mathbf{\Phi}}(k-1) + \frac{\eta \mathbf{\Delta U}(k-1)(\Delta y(k) - \hat{\mathbf{\Phi}}(k-1)^T \mathbf{\Delta U}(k-1))}{\mu + \|\mathbf{\Delta U}(k-1)\|^2}; \\ &\hat{\mathbf{\Phi}}(k) = \hat{\mathbf{\Phi}}(1), \quad if\ \|\hat{\mathbf{\Phi}}(k)\| \leq \tilde{\epsilon}\ or\ sign(\hat{\phi}_1(k)) \neq sign(\hat{\phi}_1(1)) \end{aligned} \tag{6}$$

with $\Delta y(k)$ reading as in (5) and $\eta \in (0,2)$, $\tilde{\epsilon} > 0$.

Remark 2.2 ([5]). Without loss of generality, it is here assumed that $\hat{\phi}_1(k) > 0$ in view of (6).

3. Problem statement

The control problem addressed in this paper is the output regulation to a constant reference variable y^*. As usual, the tracking error is defined as $e(k) \triangleq y(k) - y^*$. Extending the proof reported in [5], the boundedness of the PPD estimated parameters can be shown, even in the case of possible loss of effectiveness.

Corollary 3.1. *It is given the plant (5) subject to loss of actuator effectiveness according to (3). The estimated parameters given by (6) are norm bounded.*

Proof. The statement is an extension of the proof of Th.2 in [5]. Indeed, following the lines of the cited proof, defining the estimation error $\tilde{\mathbf{\Phi}}(k) \triangleq \hat{\mathbf{\Phi}}(k) - \mathbf{\Phi}(k)$, it holds:

$$\|\tilde{\mathbf{\Phi}}(k+1)\| \leq d_1 \|\tilde{\mathbf{\Phi}}(k)\| + \|\mathbf{\Theta}(k)\mathbf{\Phi}(k)\| + \|\mathbf{\Phi}(k-1)\| \tag{7}$$

with

$$\mathbf{\Theta}(k) = \left[\mathbf{I} - \eta \frac{\mathbf{\Delta U}(k)\mathbf{\Delta U}(k)^T}{\mu + \|\mathbf{\Delta U}(k)\|^2}(\mathbf{I} - \mathbf{\Delta}(k))\right] \mathbf{\Phi}(k) \tag{8}$$

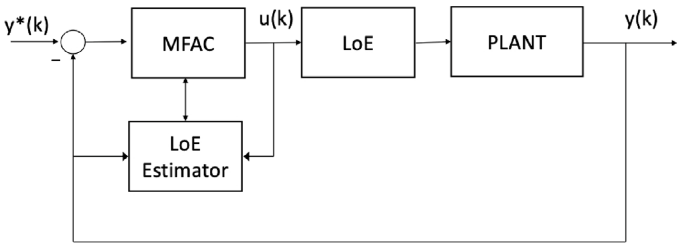

Fig. 1. Block scheme of the control architecture.

with $0 < d_1 < 1$. Considering the squared norm of $\mathbf{\Theta}(k)$ it can be proved that, in view of $\|\mathbf{\Delta}(k)\| \le 1$, $\|\mathbf{I} - \mathbf{\Delta}(k)\| \le 1$ and of the choice $\eta \in (0, 2)$ one gets, after some manipulations:

$$\begin{aligned} \|\mathbf{\Theta}(k)\|^2 &< \|\mathbf{\Phi}(k)\|^2 - 2\eta \frac{\mathbf{\Phi}(k)^T \mathbf{\Delta}(k)\mathbf{\Delta U}(k)\mathbf{\Delta U}(k)^T(\mathbf{I} - \mathbf{\Delta}(k))\mathbf{\Phi}(k)}{\mu + \|\mathbf{\Delta U}(k)\|^2} \\ &< \left(1 + 2\frac{\|\mathbf{\Delta U}(k)\|^2}{\mu + \|\mathbf{\Delta U}(k)\|^2}\right)\|\mathbf{\Phi}(k)\|^2 \stackrel{\triangle}{=} \alpha^2 \|\mathbf{\Phi}(k)\|^2 \end{aligned} \tag{9}$$

where α can be made arbitrarily close to unity by the parameter μ. Therefore, according to Theorem 2.1, the boundedness of the estimation error follows since

$$\|\tilde{\mathbf{\Phi}}(k+1)\| \le d_1 \|\tilde{\mathbf{\Phi}}(k)\| + b(1+\alpha) \tag{10}$$

and

$$\|\tilde{\mathbf{\Phi}}(k+1)\| \le d_1^k \|\tilde{\mathbf{\Phi}}(0)\| + \frac{b_1(1+d_1^k)}{1-d_1} \tag{11}$$

with $b_1 \stackrel{\triangle}{=} b(1+\alpha)$. It follows that $\|\hat{\mathbf{\Phi}}(k)\| \le \hat{b}$ for some $\hat{b}$, since $\|\tilde{\mathbf{\Phi}}(k)\| \le \tilde{b}$ for a finite $\tilde{b}$ from (11) and $\|\mathbf{\Phi}(k)\|$ is bounded from Theorem 2.1. △

4. Resilience enhancement to actuator faults

The previous considerations can be used to define a sufficient condition allowing the detection of the occurred actuator LoE, as well as a method for improving the fault-resiliency of the controller. Consider the plant (5) satisfying Assumption 2.1, possibly affected by a LoE according to (3). The following controller is introduced (see Fig. 1), obtained by a simple modification of the original control law proposed in [5]:

$$\Delta u(k) = \frac{\hat{\phi}_1(k)}{\gamma + |\hat{\delta}(k)|} \cdot \frac{\rho_1 e(k) - \sum_{i=2}^{d_u} \rho_i \hat{\phi}_i(k) \Delta u(k-i+1)}{\lambda + \hat{\phi}_1(k)^2} \tag{12}$$

with $\lambda, \gamma > 0$ to be defined, $\rho_i \in (0, 1]$, $i = 1, \dots, d_u$. The novelty here is the presence of the estimated LoE $\hat{\delta}(k)$, $k > d_u + 1$, given by

$$\hat{\delta}(k) = \hat{\delta}(k - d_u - 1) + \begin{cases} \eta_\delta \frac{\hat{\phi}_1(k-1)\Delta u(k-1)}{\mu + \hat{\phi}_1(k-1)^2 \Delta u(k-1)^2} \cdot \\ \cdot \left[\Delta y(k) - \sum_{i=2}^{d_u} \hat{\phi}_i(k-1)\hat{\delta}(k-i+1)\Delta u(k-i)\right]; \\ \hat{\delta}(k-1) \quad if \;\; |\Delta u(k-1)| < \epsilon_\delta, \quad \epsilon_\delta > 0; \\ \delta_m \quad if \;\; |\hat{\delta}(k)| < \delta_m; \\ 1 \quad\;\; if \;\; |\hat{\delta}(k)| > 1 \end{cases} \tag{13}$$

with arbitrary $\mu > 0$, and with $\eta_\delta \ge 1$ being an arbitrary finite amplification factor. Features and closed loop convergence of the proposed control scheme is investigated in the following result:

Theorem 4.1. *Consider the plant (5), with the actuator possibly affected by a LoE (3) satisfying Assumption 2.1. The system is assumed fed by (12), (13). There exists $\lambda_{min} > 0$, $\gamma, \mu > 0$ such that for $\lambda > \lambda_{min}$ the closed loop tracking error vanishes asymptotically. Moreover, the estimation error of (13) is bounded.*

Proof. The proof is made of two steps. Boundedness of the norm of the control input $\|\mathbf{\Delta U}(k)\|$ will be shown first, based on the proof of Th.2 in [5] along with the asymptotic vanishing of the tracking error. Next, boundedness of the estimation error $\hat{\delta}(k) - \delta(k)$ will be proved.

Step 1. Due to the boundedness of $|\hat{\delta}(k)|$ in view of (13), there exists finite $\gamma > 0$ such that

$$\epsilon_m < \frac{1}{\gamma + |\hat{\delta}(k)|} < \epsilon_M, \quad \forall k \tag{14}$$

for suitable $0 < \epsilon_m < \epsilon_M$. In fact, γ should satisfy

$$\begin{cases} \epsilon_M(\gamma + |\hat{\delta}(k)|) > 1 & \to \gamma > \frac{1}{\epsilon_M} \\ \epsilon_m(\gamma + |\hat{\delta}(k)|) < 1 & \to \gamma < \frac{1}{\epsilon_m} - \max_k |\hat{\delta}(k)| \end{cases} \tag{15}$$

Therefore it is enough to select:

$$\begin{cases} \epsilon_M > \frac{\epsilon_m}{1-\epsilon_m} \\ \frac{1}{2} < \epsilon_m < 1 \end{cases} \tag{16}$$

Therefore, exploiting the arbitrariness of $\rho_i \in (0,1]$, $i = 1, \ldots, d_u$, and since $|\delta(k)| \leq 1$, it is easy to see that the original development of the proof of Th.2 in [5] still holds, ensuring that there exists a $\lambda_{min} > 0$ such that the vanishing of the tracking error and, in turn, the boundedness of the control input is guaranteed for λ satisfying $\lambda > \lambda_{min}$.

Step 2. Iterating backward the expression (13) up to an argument satisfying $k - Bd_u < k - \bar{k}$, with B being a suitable positive integer and $\bar{k}$ the fault occurrence time, one gets:

$$\hat{\delta}(k) = \hat{\delta}(k - B\cdot\bar{d}_u) + \sum_{j=1}^{B} \eta_\delta \frac{\hat{\phi}_1(k-(j-1)\bar{d}_u - 1)\Delta u(k-(j-1)d_u - 1)}{\mu + \hat{\phi}_1(k-(j-1)\bar{d}_u - 1)^2 \Delta u(k-(j-1)\bar{d}_u - 1)^2} \cdot$$
$$\cdot \left[\Delta y(k-(j-1)\bar{d}_u) - \sum_{i=2}^{d_u} \hat{\phi}_i(k-(j-1)\bar{d}_u - 1)\cdot \hat{\delta}(k-(j-1)\bar{d}_u - i + 1)\Delta u(k-(j-1)\bar{d}_u - 1) \right] \tag{17}$$

where $\bar{d}_u \triangleq d_u + 1$.

Let us consider now the estimation error: $\tilde{\delta}(k) \triangleq \hat{\delta}(k) - \delta(k)$. Adding and subtracting $\delta(k)$ from (13) one gets, defining $z_j \triangleq (j-1)\bar{d}_u$:

$$\tilde{\delta}(k) = \sum_{j=1}^{B} \Bigg\{ \eta_\delta \frac{\hat{\phi}_1(k-z_j-1)\Delta u(k-z_j-1)}{\mu + \hat{\phi}_1(k-z_j-1)^2 \Delta u(k-z_j-1)^2} \cdot \Bigg[-\sum_{i=2}^{d_u} \hat{\phi}_i(k-z_j-1)\tilde{\delta}(k-z_j-i+1)\Delta u(k-z_j-i) +$$
$$\phi_1(k-z_j-1)\delta(k-z_j-1)\Delta u(k-z_j-1) + \sum_{i=2}^{d_u} \big[\phi_i(k-z_j-1)\delta(k-z_j-i) -$$
$$\hat{\phi}_i(k-z_j-1)\delta(k-z_j-i+1)\big]\, \Delta u(k-z_j-i) \Bigg] \Bigg\} + \hat{\delta}(k - B\cdot\bar{d}_u) - \delta(k). \tag{18}$$

In view of Assumption 2.1:

$$\tilde{\delta}(k) = \sum_{j=1}^{B} \Bigg\{ \eta_\delta \frac{\hat{\phi}_1(k-z_j-1)\Delta u(k-z_j-1)}{\mu + \hat{\phi}_1(k-z_j-1)^2 \Delta u(k-z_j-1)^2} \cdot \Bigg[-\sum_{i=2}^{d_u} \hat{\phi}_i(k-z_j-1)\tilde{\delta}(k-z_j-i+1)\Delta u(k-z_j-i) +$$
$$\phi_1(k-z_j-1)\delta(k-z_j-1)\Delta u(k-z_j-1) - \sum_{i=2}^{d_u} \tilde{\phi}_i(k-z_j-1)\delta(k-z_j-i)\Delta u(k-z_j-i) \Bigg] \Bigg\} +$$
$$\hat{\delta}(k - B\cdot\bar{d}_u) - \delta(k) \tag{19}$$

Define the term appearing as a common factor in the previous expression as follows:

$$\varphi(k-1) \triangleq \frac{\hat{\phi}_1(k-1)\Delta u(k-1)}{\mu + \hat{\phi}_1(k-1)^2 \Delta u(k-1)^2}. \tag{20}$$

Since all the variables are bounded it holds $\forall k > 1$:

$$|\varphi(k-1)| \leq \frac{1}{2\sqrt{\mu}};$$
$$|\hat{\phi}_i(k-1)\varphi(k-1)| \leq \frac{\hat{b}}{2\sqrt{\mu}} \triangleq \hat{M};$$
$$|\tilde{\phi}_i(k-1)\varphi(k-1)| \leq \frac{\tilde{b}}{2\sqrt{\mu}} \triangleq \tilde{M};$$
$$|\phi_1(k-1)\varphi(k-1)| \leq \frac{b}{2\sqrt{\mu}} \triangleq M; \tag{21}$$

therefore:

$$|\tilde{\delta}(k)| \leq \eta_\delta \hat{M} \left[\; |\tilde{\delta}(k-1)| \quad |\tilde{\delta}(k-2)| \quad \ldots \quad |\delta(k - Bd_u + 1)| \; \right] \cdot$$

$$\cdot \begin{bmatrix} |\Delta u(k-2)| \\ |\Delta u(k-3)| \\ \vdots \\ |\Delta u(k-Bd_u)| \end{bmatrix} + \eta_\delta \max(\tilde{M}, M) B \|\mathbf{\Delta}\bar{\mathbf{U}}(k-1)\| + 1 + |\,\hat{\delta}(k - B \cdot d_u)| \tag{22}$$

with $\|\mathbf{\Delta}\bar{\mathbf{U}}(k-1)\| \stackrel{\triangle}{=} \|\mathbf{\Delta U}(k-1)\| + \|\mathbf{\Delta U}(k-d_u-1)\| + \cdots + \|\mathbf{\Delta U}(k-(B-1)d_u-1)\|$. It follows that $\|\mathbf{\Delta}\bar{\mathbf{U}}(k-1)\| \leq \sum_{i=1}^{Bd_u} |\Delta u(k-i)|$ which is bounded according to the final step of the proof of Theorem 2 in [5]. The selection of the integer B ensures that $\hat{\delta}(k - B \cdot d_u) = \delta(k - B \cdot d_u) = 1$ (when the fault has not yet occurred), therefore the difference equation describing the dynamics which bounds the estimation error is linear and reads

$$|\tilde{\delta}(k)| - c_1 \left[|\tilde{\delta}(k-1)| + |\tilde{\delta}(k-2)| + \cdots \right] = K^*, \tag{23}$$

where $c_1 \stackrel{\triangle}{=} \eta_\delta \hat{M} \|\mathbf{\Delta}\bar{\mathbf{U}}(k-1)\|$ can be made arbitrarily small by a proper selection of the parameter μ in $\hat{M}$ due to (21), and K^* is a positive constant. The polynomial in the z^{-1} variable associated to the difference Eq. (23) tends to $z^{Bd_u} = 0$ for $c_1 \to 0$, therefore, exploiting the continuity of its roots, it can be argued that they can be confined inside the unit circle in the z-plane for suitably chosen parameters. With this choice, the step response of the linear discrete-time system (23) is inherently bounded, and it follows that the estimation error $|\tilde{\delta}(k)|$ is bounded $\forall k$. △

Remark 4.1. As argued in [1], the main shortcoming of data-driven Model Free Controllers is the difficult systematic stability and robustness analyses, mainly because these analyses require detailed mathematical models of the controlled process. The previous mathematical development, coupled with the fact that in the considered MFAC framework the dynamic linearized system does not lose any information of the nonlinear system therefore is its equivalent [5,13], proves that the closed loop system is externally stable.

Remark 4.2. It is important to consider that after the occurrence of a fault (3), the vanishing of the tracking error is no longer guaranteed by the standard algorithm. Indeed, system variables are likely to temporarily increase while the estimation algorithm tries to account for the faulty condition, as the vanishing of the error $\hat{\mathbf{\Phi}}(k) - \delta(k)\mathbf{\Phi}(k)$ is inherently pursued by (6) due to its adaptive character. Though the data-driven controller could be able to partially accommodate the occurred LoE exploiting its adaptivity (but there is no guarantee), an undesired, potentially wide, transient phase is expected while the faulty condition is attempted to be compensated. The aim of the following development is to timely detect the fault and reduce, as much as possible, this undesired transient.

It might be likely to assume that the fault occur after the extinction of the initial transient phase of the adaptive controller previously introduced, i.e., at an (unknown) time instant $\bar{k} > 0$ such that $e(\tau) \simeq 0$ for $k_1 < \tau < \bar{k}$, for some $k_1 < \bar{k}$. Under these circumstances, a straightforward condition can be given for fault detection:

Lemma 4.1. *Consider the plant (2) possibly affected by a loss of actuator effectiveness (3) fulfilling the previous assumption, and driven by (12). Define by $\tau < \bar{k}$ a time instant when the transient phase can be considered as extinguished. If for some $k > \tau$ it holds:*

$$\left| \frac{\hat{\phi}_i(k-1)}{\hat{\phi}_i(\tau)} - 1 \right| > \epsilon_{tol} \tag{24}$$

for an arbitrary integer $i \in [1, d_u]$, and for a fixed tolerance $\epsilon_{tol} > 0$, then a LoE is occurring at the actuator at the time instant $k-1$.

Proof. The statement derives directly from the form of the faulty plant (5) and the result of Corollary 3.1. △

5. Simulation studies

In order to underpin the theoretical development presented with simulation data, a classic benchmark system was used for testing. The system is the flexible transmission system proposed in [3] as a benchmark for the design of digital controllers in the original paper on IFT. Interestingly, the same system was reconsidered in [4] (in the unloaded case) with reference to the VRFT method. Since both IFT and VRFT belong to the offline DDC class, it might be interesting to treat the same benchmark system with online DDC design methods. For a comprehensive comparison of the online adaptive methods, both the original MFAC scheme [5] and the recent technique [28], which couples MFAC and Prescribed Performance Control, are considered here, in addition to the proposed approach.

The flexible transmission system consists of three horizontal pulleys connected by two elastic belts. The input and output of the system are the angular position of the first and third pulleys respectively. The control objective is output regulation to zero (details about the system and the additional control requirements used for each specific technique can be found in the references cited). In the unloaded case, the considered system results in a non-minimum phase system whose discretized expression with a sampling time $T_c = 0.05$ s is given in [4].

The novel perturbation here is the presence of a severe, abrupt loss of effectiveness of the form (3) affecting the actuator, with $\bar{\delta} = 0.2$ starting from $\bar{k} = 300$. For the controller presented here, tests were carried out with the following parameters: $d_u = 4$, $\mu = 10$, $\lambda = 60$, $\hat{\mathbf{\Phi}}(0) = [2\ 0.1\ 0.1\ 0.1]^T$. Occurrence of the fault was detected at $k = 318$ (detection was inhibited during an initial transient

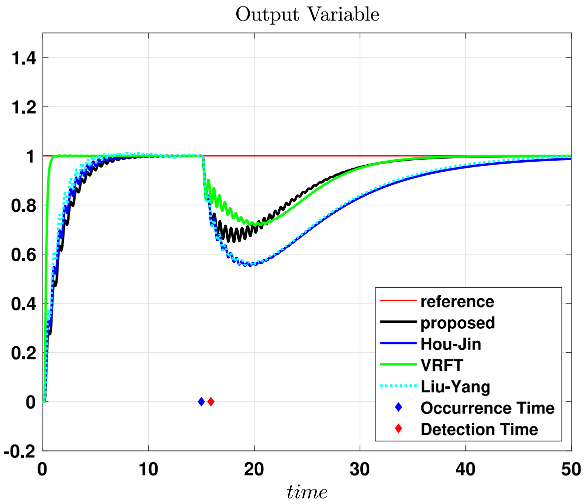

Fig. 2. Output variable: comparative plot.

Table 1
IAE of the tracking error.

Removed transient (samples)	Proposed	Hou-Jin 2011	VRFT	Liu-Yang 2019
100	63.2386	122.7807	57.72584	118.1924
400	34.6070	88.0862	37.4457	83.3908

phase of 150 samples ($7s$)), and the identification/accommodation algorithms (13), (12) were started at this time, with $\mu_\delta = 0.02$, $\gamma = 0.15$, $\hat{\delta}(0) = 0.5$, $\epsilon_{tol} = 0.001$. The comparative performance analysis, which in addition to the controller reported in [4], case 2 (no noise) also shows the controllers presented in [5,28], can be seen in Fig. 2 showing the output variable. For the algorithm [5] the same setting reported above was kept, while the controller [28] was set as follows (referring to the original notation of the paper): $\Gamma = 0.01$, $\alpha = 0.5$, $\lambda = 60$, $\theta_\rho = 0.05$, $\rho_\infty = 0.01$. Further simulation results are shown in Fig. 3 (control input), Fig. 4 (estimated fault amplitude), Fig. 5 (Parameters $\boldsymbol{\Phi}$). The tracking performances are summarized in Table 1, where an initial transient of a number of samples reported in the first column was removed to exclude the warming-up of the adaptive mechanism.

5.1. Discussion

Based on the reported simulation results, a few considerations are in order. The tracking performances measured by the Integral of the Absolute Error (IAE), listed in the first row of Table 1, show a superiority of the VRFT controller, mostly attributable to the very good initial transient compared to online techniques. This is not surprising given the inherent characteristics of offline DDCs compared to their online counterparts as described in the introduction. If we restrict our attention to the response to the abrupt fault (second row of Table 1), the proposed approach seems to outperform the rest of the approaches in terms of measured performance. Again, this is an expected result, since the algorithm presented here is the only one that has a dedicated mechanism for managing faults, while the other techniques rely only on their inherent robustness/adaptivity to deal with the unexpected loss of effectiveness. As for the practical use of the proposed controller, it shares the well-known advantages and disadvantages of the class of adaptive DDC to which it belongs. Namely, a potential weakness is the possible dependence on initial conditions and the need for careful parameter tuning. Conversely, no pre-recorded input/output data are required, nor does a predetermined controller structure need to be defined a priori. The most interesting merits of the proposed approach, tied to the MFAC framework of which it is part, are perhaps the broad class of plants it can address (no identification of a specific plant model is performed due to the use of PPDs), as well as the very limited computational effort.

6. Conclusions

Inspired by the seminal papers on the MFAC approach [5,12] in the context of DDC, this work investigated the controller resiliency enhancement in case of actuator LoE for a SISO system. A fault estimation algorithm was proposed, with bounded estimation error, as well as a condition for detecting faults based on the very structure of the controlled plant. In addition, the redesign of a well known tracking control algorithm was proposed that ensures resiliency with respect to the actuator LoE. To support the technical development, a comparative simulation study was also reported, carried out referring to a benchmark plant taken from the DD literature and already used for testing purposes. The results have shown that the inherent adaptation properties

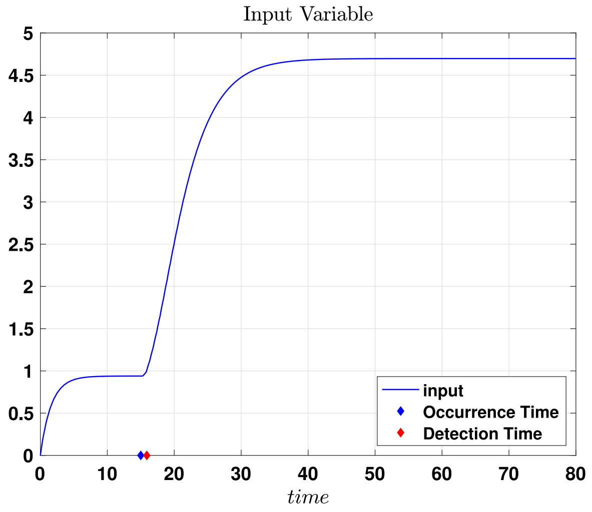

Fig. 3. Control input.

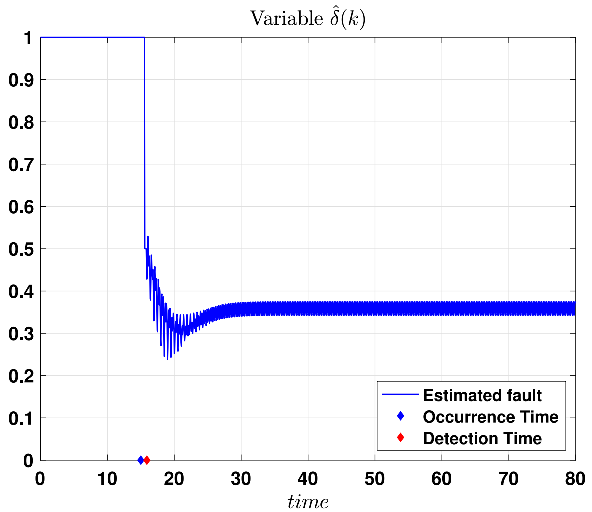

Fig. 4. Estimated variable $\hat{\delta}(k)$.

of MFAC controllers can be enhanced, leading to performance improvements in terms of control authority and tracking accuracy. Future work will consider the extension of the presented approach to the Full Form Dynamic Linearization data model and to MIMO plants.

CRediT authorship contribution statement

Maria Letizia Corradini: Writing – review & editing, Writing – original draft, Validation, Software, Methodology, Formal analysis, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

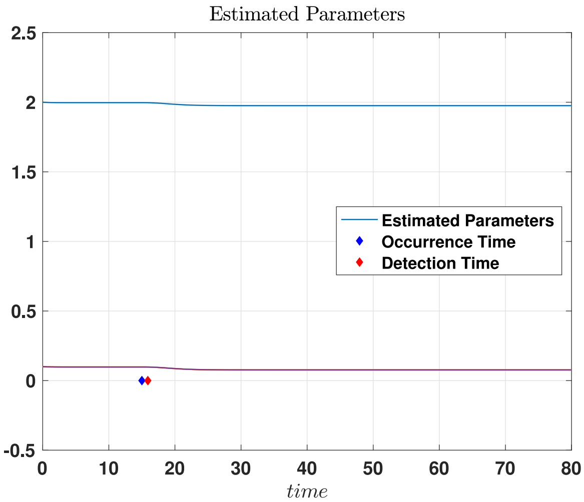

Fig. 5. Estimated parameters $\hat{\Phi}(k)$.

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